

# Dynamic simulation of deployable structures based on discrete variational method

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**Abstract:** The expansion process of negative Poisson's ratio structures was analyzed using multi body dynamics methods. A multi body system dynamics model was established using Cartesian coordinates for the deployable structure, and the dynamic equations of the expansion process were obtained. The discrete variational method of multi body system dynamics based on center of gravity interpolation was applied for numerical solution. The experimental results verified that the discrete variational method has the characteristics of high accuracy and accurately maintaining the constraint equation when the step size is large.

**Keywords:** Deployable structure; Multibody system; Dynamic simulation; Discrete variational method

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## 1. Introduction

A deployable structure is a structural system that satisfies the Malwell criterion through finite mechanism displacement or infinitesimal structural displacement, and has two stable configurations: a fully folded state and a fully unfolded state [1]. The fully folded structure has a small volume and is easy to transport and store; Fully unfolded state, with structural expansion that can cover three-dimensional space and maintain normal operational working state; There is a continuous and smooth motion path between the folded and unfolded states [2].

In 1960, Buckminster Fuller first proposed the concept of deployable structures. On this basis, Spanish architect E.P. Pinero applied the deployable structure to his design work - "Mobile Theater" [3]. In the late 1980s, the research level of deployable structure theory and technology applications rapidly improved in the aerospace field, including NASA, ESA, Cambridge University, Massachusetts Institute of Technology, Oxford University, and others who proposed various concepts of deployable structure systems. China gradually began relevant research and exploration in the late 1990s, with numerous research institutions such as Zhejiang University conducting extensive research on the design principles, structural configurations, and mechanism motion analysis of deployable structures, achieving fruitful results [4].

The division of deployable structures follows different standards and varies in different fields. According to the 1998 International Association of Shell and Space Structures, deployable structures are conceptually divided into foldable structures, open and close structures, tensioned integral structures, inflatable structures, and cable membrane structures [5]. Among them, folding structures can be subdivided into rigid deployable structures, shear hinge structures [6], rigid origami structures [7], and climbing domes [8]. The opening and closing structure can be divided into opening bridge structure and opening and closing roof structure.

Structural unfolding is a process of zero strain, large displacement, and uncertainty, and traditional structural theory cannot accurately study it. Therefore, it is necessary to introduce mechanism theory. At present, the modeling methods for deployable structures mainly include multi body dynamics method [9], force method, generalized inverse method [10], finite element method [11], Lie group method [12], etc.

According to the specific design requirements of deployable structures, the structure is generally optimized from the following three types of objectives. (1) Minimum unfold and fold time: used to optimize the unfold and fold time of deployable structures to improve their response speed and application range. (2) Maximum unfolding and folding stiffness: used to optimize the unfolding and folding stiffness of deployable structures to ensure their stability and reliability. (3) Minimum unfolding and folding energy consumption: used to optimize the unfolding and folding energy consumption of deployable structures, in order to reduce their usage costs and environmental performance.

This article uses the multi body dynamics method to analyze the unfolding process of deployable structures. A multi body system dynamics model is established using Cartesian coordinates for the deployable structure, and the dynamic equations of the unfolding process are obtained. The discrete variational method and Runge-Kutta method are used for numerical solution, and experiments have verified that the discrete variational method has the characteristics of high accuracy and accurately maintaining the constraint equation when the step size is large.

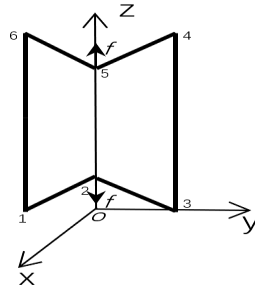
## 2. Modeling of deployable structures

This article analyzes the characteristics of a single deployable element for a space extended circular antenna and obtains a simplified physical model. As the main body of the structure is composed of deployable elements, the main consideration for modeling is the deployable element. In addition, the main components of deployable structures are rigid members, without considering the deformation of the members.

Using Cartesian coordinate modeling [13], with the center of the deployable unit as the coordinate origin, establish a Cartesian coordinate system as shown in Figure 1, and number the hinge points of the deployable unit counterclockwise from bottom to top.

The generalized coordinates of the entire scissor element are

$$\mathbf{q} = (x_1 \ y_1 \ z_1 \ x_2 \ y_2 \ z_2 \ x_3 \ y_3 \ z_3 \ x_4 \ y_4 \ z_4 \ x_5 \ y_5 \ z_5 \ x_6 \ y_6 \ z_6)^T \tag{1}$$



**Figure 1. Deployable unit**

The deployable element consists of six spatial truss elements (rods 12, 23, 34, 41, 56, 67, etc. in Figure 1). In the overall coordinate system, the consistent mass matrix of the spatial truss elements is

$$\mathbf{M}^e = \iiint_{V^e} \rho \mathbf{N}^T \mathbf{N} dV = \frac{\rho A l}{6} \begin{pmatrix} 2 & & & & & \\ 0 & 2 & & & & \\ 0 & 0 & 2 & & & \\ 1 & 0 & 0 & 2 & & \\ 0 & 1 & 0 & 0 & 2 & \\ 0 & 0 & 1 & 0 & 0 & 2 \end{pmatrix} \tag{2}$$

Among them,  $\rho$  is the density of the truss,  $A$  is the cross-sectional area of the truss,  $l$  is the length of the truss, and matrix  $\mathbf{N}$  is the shape function.

$$\mathbf{N} = \begin{pmatrix} \frac{l-x}{l} & 0 & 0 & \frac{x}{l} & 0 & 0 \\ 0 & \frac{l-x}{l} & 0 & 0 & \frac{x}{l} & 0 \\ 0 & 0 & \frac{l-x}{l} & 0 & 0 & \frac{x}{l} \end{pmatrix} \tag{3}$$

The node displacement component of rod 12 in the overall coordinate system is

$$\mathbf{X}_{12} = (x_1 \ y_1 \ z_1 \ x_2 \ y_2 \ z_2)^T \tag{4}$$

The kinetic energy of shear rod 12 is

$$T_{12} = \frac{1}{2} \dot{\mathbf{X}}_{12}^T \mathbf{M}_{12} \dot{\mathbf{X}}_{12} \tag{5}$$

Substitute equations (2) and (4) into equation (5) to obtain

$$T_{12} = \frac{\rho A l_1}{6} (\dot{x}_1^2 + \dot{x}_1 \dot{x}_2 + \dot{x}_2^2 + \dot{y}_1^2 + \dot{y}_1 \dot{y}_2 + \dot{y}_2^2 + \dot{z}_1^2 + \dot{z}_1 \dot{z}_2 + \dot{z}_2^2) \tag{6}$$

Among them,  $l_1$  is the length of rod 12. The kinetic energy of rods 23, 34, 45, 56, and 61 is the same.

The total kinetic energy of the entire deployable unit is

$$T = T_{12} + T_{23} + T_{34} + T_{45} + T_{56} + T_{61} \tag{7}$$

The Lagrange function is

$$L = T - V \tag{8}$$

The non potential generalized force acting on the entire deployable element is

$$\mathbf{F} = (0 \ 0 \ 0 \ 0 \ 0 \ -f \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ f \ 0 \ 0 \ 0)^T \tag{9}$$

Under the Lagrange mechanics system, the following Lagrange equation can be derived from the principle of virtual work and the principle of d'Alembert

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}} \right) - \frac{\partial L}{\partial q} = F \tag{10}$$

Substitute equations (8) and (9) into equation (10) to obtain the differential equation

$$M\ddot{q} = F \tag{11}$$

The generalized coordinates of deployable elements are not independent of each other during the unfolding process, and they need to satisfy some constraint conditions, including position constraint  $\Phi_1$  and rod length constraint  $\Phi_2$ . The constraint equation

$$\Phi = \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} = 0 \tag{12}$$

Differential equations with Lagrange multipliers can be obtained

$$M\ddot{q} + \Phi_q^T \lambda = F \tag{13}$$

The following system of differential algebraic equations is obtained

$$\begin{cases} M\ddot{q} + \Phi_q^T \lambda = F \\ \Phi = 0 \end{cases} \tag{14}$$

The dynamic equation of the unfolding process is called the Euler Lagrange equation.

### 3. Numerical examples

Using the discrete variational method, numerically solve the dynamic equations of the unfolding process. The discrete variational method is based on the Hamiltonian principle. Firstly, equidistant nodes are selected for Lagrangian interpolation in time  $t$ , and the state variable and its derivatives are discretized in the time domain. Then, the Gauss Legendre quadrature formula and the discrete variational principle are used to obtain the discretized Euler Lagrange equation. Finally, the discrete value of the state variable is obtained by solving the algebraic equation system, which is compared to the traditional Runge Kutta method. This method is suitable for large step lengths and has high accuracy.

For a fully constrained multibody system, its Hamiltonian action can be expressed as

$$S = \int_0^T (L(q, \dot{q}, t) - \lambda^T \Phi(q, t)) dt \tag{15}$$

Among them,  $\lambda \in R^m$  is the Lagrange multiplier, and  $\Phi(q, t) \in R^m$  is the complete constraint.

In the numerical simulation of multi body system dynamics, the discrete variational method first discretizes the Hamiltonian action, and then uses the discrete variational principle to obtain the discrete Euler Lagrange equation.

Divide the simulation time  $[0, T]$  into several intervals  $[t_i, t_{i+1}]$ ,  $i=0, 1, \dots, N$ . Assuming  $q_d(t), \dot{q}_d(t)$  is the discrete form of generalized coordinates and generalized velocities on the interval  $[t_i, t_{i+1}]$ , the discrete Hamiltonian action is

$$\begin{aligned} S_d &= \sum_{i=0}^N \int_{t_i}^{t_{i+1}} (L(q_d, \dot{q}_d, t) - \lambda^T \Phi(q, t)) dt \\ &= \sum_{i=0}^N \left( \sum_{j=0}^M A_j L(q_d(t_{ij}), \dot{q}_d(t_{ij}), t_{ij}) - \lambda_i^T \Phi(q_i, t_i) \right) \end{aligned} \tag{16}$$

Where  $A_j, j=0, 1, \dots, M$  is the numerical integration coefficient,  $t_{ij}$  is the quadrature node of the interval  $[t_i, t_{i+1}]$ ,  $i=0, 1, \dots, N, j=0, 1, \dots, M$ .

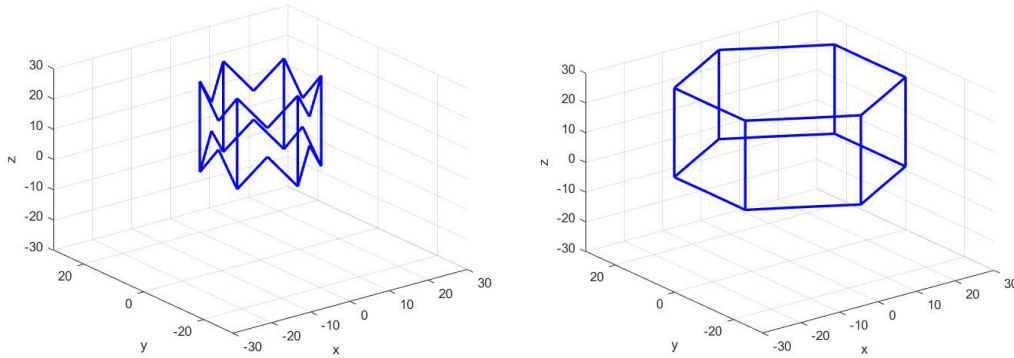
Usually,  $q_d(t), \dot{q}_d(t)$  can be expressed as interpolation functions with function values  $q_{i,1} \dots q_{i,s}$  at several points in  $[t_i, t_{i+1}]$ , such as Lagrange interpolation polynomials. By using the principle of discrete variation, the extreme value of the discrete Hamiltonian action functional variation can be obtained to obtain the discrete Euler Lagrange equation for  $q_{i,1} \dots q_{i,s}, i = 0, 1, \dots, N$ , which is a discrete variational mathematical model of multi body system dynamics.

$$\begin{cases} \sum_{j=1}^M A_j \left( D_1 L(q_{i,1}, \dots, q_{i,s}, t_{i,j}) + D_s L(q_{i-1,1}, \dots, q_{i-1,s}, t_{i-1,j}) \right) - \Phi_q^T(q_i, t_i) \lambda_i = 0 & i = 0, 1, \dots, N \\ \sum_{j=1}^M A_j D_k L(q_{i,1}, \dots, q_{i,s}, t_{i,j}) = 0, k = 2, \dots, s-1 \\ \Phi(q_i, t_i) = 0 \end{cases} \tag{17}$$

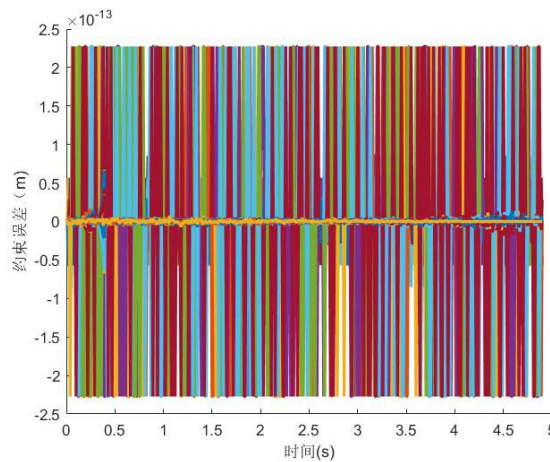
By using initial conditions  $q(0)$ ,  $\dot{q}(0)$ , the equation can be solved to obtain generalized coordinate discrete values, thus achieving numerical simulation of multi body system dynamics.

**4. Numerical examples**

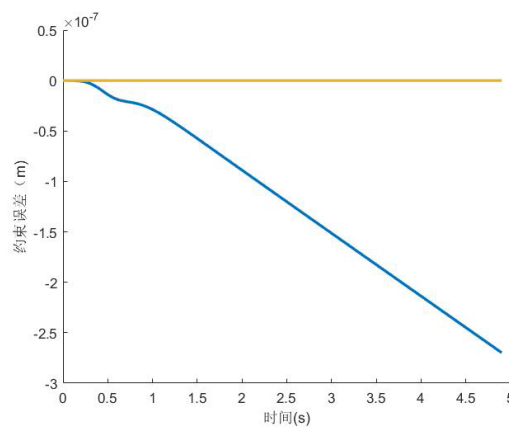
Using numerical solution methods, simulation experiments were conducted on a model containing 6 deployable elements as an example. Rod density=3000 kg/m<sup>3</sup>, cross-sectional area  $A=0.03 \times 0.03$  m<sup>2</sup>, with a horizontal rod length of  $l_1=3$  m, a vertical rod length of  $l_2=1.2$  m, and a driving force of  $f=200$  N. The time step is  $h=0.01$ s, and the simulation results are shown in Figures 3.1 to 3.3. The unfolding process is 4.92s.



**Figure 2. Configuration diagram at each time point**



**Figure 3. Constraint Error Diagram of Discrete Variational Method**



**Figure 4. Runge Kutta method constraint error diagram**

The configuration diagram of the deployable structure at each moment during the unfolding process is shown in Figure 3.1. From Figures 3.2-3.3, it can be seen that when the time steps are all taken as 0.01s, the order of magnitude of the constraint error corresponding to the discrete variational method is significantly lower than the traditional fourth order Runge Kutta method.

The discrete variational method can accurately maintain the constraint equation under large step sizes, as shown in Tables 1 and 2. The time steps are taken as 0.01s and 0.005s respectively, where  $t_{cpu}$  represents the running time and  $\varepsilon(\Phi)$  represents the maximum error of the system constraint function.

**Table 1. Comparison of Results between Discrete Variational Method and Runge Kutta Method,  $h=0.01s$**

Method	$t_{cpu}$	$\varepsilon(\Phi)$
Discrete Variational Method	52.3906	$2.2737 \times 10^{-13}$
Runge Kutta Method	22.5313	$2.6980 \times 10^{-7}$

**Table 2. Comparison of Results between Discrete Variational Method and Runge Kutta Method,  $h=0.005s$**

Method	$t_{cpu}$	$\varepsilon(\Phi)$
Discrete Variational Method	108.25	$4.7368 \times 10^{-11}$
Runge Kutta Method	43.5781	$1.6918 \times 10^{-8}$

From Tables 1 and 2, it can be seen that under the same step size, the maximum constraint error of the discrete variational method is significantly better than that of the Runge Kutta method. It can be further observed that the maximum constraint error of the system decreases when the step size is changed from 0.005 to 0.01. The experiment verifies that the discrete variational method has the characteristics of high accuracy and accurately maintaining the constraint equation when the step size is large.

## 5. Conclusion

The multi body dynamics method was used to analyze the unfolding process of deployable structures, and Cartesian coordinate modeling was used to obtain the dynamic equations of the unfolding process. The discrete variational method of multi body system dynamics based on center of gravity interpolation and the traditional Runge Kutta method were used for numerical solutions respectively. The experimental results showed that the discrete variational method has the characteristics of high accuracy and accurately maintaining the constraint equation when the step size is large.

## Conflict and interest

The authors confirm that this article content has no conflict of interest.

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