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# A generalized- α projection control method for multi-agent formation preserving

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***Abstract:*** **For the formation preservation problem of multi-agent described by Lagrangian system, a generalized-α projection control method based on differential algebraic equations is proposed. Firstly, the Lagrange equation is used to model the general multi-agent system, then the geometric algebraic equation is designed to describe the formation, and the Lagrange multiplier method is used to calculate the control rate, so as to achieve the precise control of the multi-agent system formation. This method uses the idea of analytical mechanics to control the virtual structure among multiple agents by binding force, and uses the advantage of generalized-α projection method to maintain the constraint accurately to achieve accurate control. Compared with the Baumgarte stabilization method, the numerical results show that the generalized-α projection control method can avoid the difficulty of parameter selection and realize the accurate maintenance of multi-agent formation control at large strides.**

***Keywords:*** Generalized-α method, Differential-algebraic equation, Binding force, Projection method, Multi-agent formation

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## 1. Introduction

Multi-agent system is a complex dynamic coupling system composed of multiple agents, whose coupling is reflected in the information interaction between agents through the network to form a specific formation and cooperate with each other to complete the target task. Due to the wide application of multi-agent systems in mechanical systems, ships, aircraft, vehicles, spacecraft, robots and other fields, the stability control of multi-agent formation has attracted more and more attention from researchers and engineers.

In recent years, a lot of scientific achievements have been made in the cooperative control of multi-agent formation. A variety of formation control methods, such as the pilot following method[1,2], virtual structure method[3], behavior method[4] and graph theory method [5], virtual potential field method[6,7], etc., have been gradually proposed. In addition, four applications of nonlinear multi-agent formation are generated, including rigid body attitude system, independent wheeled robot system, multi-Lagrangian system and other nonlinear systems.

Among them, Lagrangian systems have attracted much attention due to their ability to describe a wide range of mechanical systems, including practical systems such as robotic arms, unmanned vehicles, UAVs, underwater vehicles, and spacecraft. For example, in recent years, the distributed spacecraft formation control algorithm based on output feedback for double-layer structure[8], the cooperative formation control algorithm for pilot satellites[9], and the adaptive coordination control of multi-manipulator system[10] have emerged for multi-Lagrangian systems. However, the above studies are based on ordinary differential equations to establish the model, and few studies start from differential algebraic equations.

In the existing differential-algebraic equation modeling[11,12,13,14], the formation control of multi-agent formation problem is realized by constructing constraint function, that is, Lagrangian term provides the reaction force to maintain the kinematic constraint, so that the multi-agent formation maintains the virtual structure. In this control method [15], the control rate of multi-agent formation is transformed into the stability problem in the differential-algebraic equation through the method of analytical mechanics, so that the multi-agent formation problem can be solved using many stabilization methods in the differential-algebraic equation [16].

There are many stabilization methods in the numerical solution of differential-algebraic equations, including[17,18,19,20]. Among them, the generalized-α projection method[20] can meet the constraints of all levels of the differential-algebraic equations to a high degree, and can be solved stably when the step size is large, with high computational efficiency.

In this paper, a generalized-α projection method for formation preservation of multi-agent formations in multi-Lagrangian systems is studied. In this paper, the problem of Marine surface ship formation is taken as a numerical example. Compared with the Baumgarte stabilization method, the accuracy of the generalized-α projection method is discussed and compared.

## 2. DAE model for multi-agent

For a general Lagrangian system, it can be expressed as:

$$\eta = [\eta_1^T, \cdots, \eta_n^T]^T, M = \begin{bmatrix} m_1 I_2 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & m_n I_n \end{bmatrix}, \quad (1)$$



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$$T = \frac{1}{2}\dot{\eta}^T \cdot M \cdot \dot{\eta}\,, L = T - U - \lambda^T C(\eta),$$
$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\eta}_i} - \frac{\partial L}{\partial \eta_i} = \tau, i = 1 \cdots n.$$

$n$ is the number of agents in a multi-agent formation. $T$, $U$ are the kinetic energy and potential energy of the whole system, $L$ is the Lagrangian of the system, $M$ is the mass matrix of the system, $\eta_i$ is the location state variable of a single agent, $C(\eta)$ is the constraint function, which represents the target formation of the system, $\tau = -\lambda^T \frac{\partial C}{\partial \eta_i}$ is the control rate.

Constraint functions at all levels of the formation of the target formation:

$$\begin{aligned} C(\eta) &= 0, \\ \dot{C}(\eta) &= W(\eta)\dot{\eta} = 0, \\ \ddot{C}(\eta) &= W(\eta)\ddot{\eta} + \dot{W}(\eta)\dot{\eta} = 0. \end{aligned} \tag{2}$$

$W(\eta) = \frac{\partial C(\eta)}{\partial \eta}$ is the Jacobi matrix of constraint functions.

Thus, the differential algebraic system of index 3 is obtained as follows:

$$\begin{cases} M(\eta, t)\ddot{\eta} + \tau = F(\dot{\eta}, \eta, t), \\ \ddot{C}(\eta) = W(\eta)\ddot{\eta} + \dot{W}(\eta)\dot{\eta} = 0. \end{cases} \tag{3}$$

The above equation is the differential algebraic equation model of multi-agent multi-Lagrange system, where $\tau = -\lambda^T \frac{\partial C}{\partial \eta_i}$ is the Lagrange multiplier term, which represents the control rate in the multi-agent formation problem. The control rate can be obtained by solving the above equation, so that the formation can be controlled. However, since the initial value of formation problem is usually incompatible with the constraint function $C(\eta)$, the above equation cannot be solved normally. Therefore, the stabilization method of the above equation, such as Baumgarte stability method, can obtain the following equation.

$$\begin{cases} M(\eta, t)\ddot{\eta} + \tau = F(\dot{\eta}, \eta, t), \\ \ddot{C}(\eta) = W(\eta)\ddot{\eta} + \dot{W}(\eta)\dot{\eta} = -K_d\dot{C}(\eta) - K_p C(\eta). \end{cases} \tag{4}$$

The above equation is the existing differential algebraic equation model of multi-agent formation control established by Baumgarte stabilization method [11], where $K_d$ and $K_p$ are method parameters. However, the selection of parameters in Baumgarte stability method has no definite quasi-measurement, and the selection of parameters affects the solution of the equation. Secondly, The Baumgarte method can't ensure accurate preservation of all levels of constraints, and the accuracy is relatively low. Therefore, other stabilization methods are needed for further precise control[17,18,19,20], such as generalized -α projection methods that accurately preserve the constraints.

## 3. Generalized -α projection control method

The generalized -α projection method is based on the generalized -α method, which uses the projection method of displacement, velocity and acceleration to their respective constraint equations, so that the constraints at all levels are better maintained.

Take the isometric step $h$ to discretize the time interval and get the time period $[t_i, t_{i+1}]$, In this time period, the general formula of the generalized -α method is as follows:

$$\begin{cases} M_{i+1-\alpha_f}\ddot{\eta}_{i+1-\alpha_m} + W^T_{i+1-\alpha_f}\lambda_{i+1-\alpha_f} - F_{i+1-\alpha_f} = 0, \\ C(\eta_{i+1}) = 0. \end{cases} \tag{5}$$

In the formula above, $\ddot{\eta}_{i+1-\alpha_m} = (1-\alpha_m)\ddot{\eta}_i + \alpha_m\ddot{\eta}_{i+1}$, $\ddot{\eta}_{i+1-\alpha_m}$ can be viewed as $\ddot{\eta}$ in $t_{i+1-\alpha_m} = (1-\alpha_m)t_i + \alpha_m t_{i+1}$ time value. $W(\eta) = \frac{\partial C(\eta)}{\partial \eta}$ is the Jacobi matrix of constraint functions, $W^T_{i+1-\alpha_f} = \left(\frac{\partial C(\eta)}{\partial \eta}\Big|_{t=i+1-\alpha_f}\right)^T$. Related formulas of state variables are as follows:

$$\begin{aligned} \dot{\eta}_{i+1-\alpha_f} &= (1-\alpha_f)\dot{\eta}_i + \alpha_f\dot{\eta}_{i+1}, \\ \eta_{i+1-\alpha_f} &= (1-\alpha_f)\eta_i + \alpha_f\eta_{i+1}, \\ \dot{\eta}_{i+1} &= \dot{\eta}_i + h[(1-\gamma)\ddot{\eta}_i + \gamma\ddot{\eta}_{i+1}], \\ \eta_{i+1} &= \eta_i + h\dot{\eta}_i + \frac{h^2}{2}[(1-2\beta)\ddot{\eta}_i + 2\beta\ddot{\eta}_{i+1}]. \end{aligned} \tag{6}$$



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The value is described as follows:

$$\alpha_m = \frac{2\rho - 1}{\rho + 1}, \alpha_f = \frac{\rho}{\rho + 1}, \rho \in [0,1],$$
$$\gamma = \frac{1}{2} + \alpha_f - \alpha_m, \tag{7}$$
$$\beta = \frac{\left(1 + \alpha_f - \alpha_m\right)^2}{4}.$$

The above method can solve the index 3 differential-algebraic equation directly, but it can only keep the displacement constraint with high accuracy, and the velocity and acceleration constraints will cause the breach of contract. In order to keep the high precision of the constraints at all levels, the projection method is considered to be used.

At time $t_{i+1}$, define the constraint manifold

$$\Omega_{\eta_{i+1}} = \{\eta_{i+1} | C(\eta_{i+1}, t_{i+1}) = 0\}. \tag{8}$$

Set $\tilde{\eta}_{i+1}$ for $\eta_{i+1} \in \Omega_{\eta_{i+1}}$ approximation, the $\tilde{\eta}_{i+1}$ to $\Omega_{\eta_{i+1}}$ projection:

$$\begin{cases} \eta_{i+1} - \tilde{\eta}_{i+1} + W^T(\eta_{i+1})\mu_{i+1} = 0, \\ C(\eta_{i+1}) = 0. \end{cases} \tag{9}$$

In the formula above, $W(\eta) = C_\eta(\eta) = \frac{\partial C(\eta)}{\partial \eta}$.

Similarly, it can be obtained by projecting to the constraint manifold of velocity level and acceleration level:

$$\begin{cases} \dot{\eta}_{i+1} - \tilde{\dot{\eta}}_{i+1} + W^T(\eta_{i+1})\nu_{i+1} = 0, \\ \dot{C}(\dot{\eta}_{i+1}, \eta_{i+1}) = 0. \end{cases} \tag{10}$$

$$\begin{cases} \ddot{\eta}_{i+1} - \tilde{\ddot{\eta}}_{i+1} + W^T(\eta_{i+1})\omega_{i+1} = 0, \\ \ddot{C}(\ddot{\eta}_{i+1}, \dot{\eta}_{i+1}, \eta_{i+1}) = 0. \end{cases} \tag{11}$$

If the $t_{i+1}$, approximation $\tilde{\eta}_{i+1}, \tilde{\dot{\eta}}_{i+1}, \tilde{\ddot{\eta}}_{i+1}$ has been calculated, can undertake constraint projection, respectively, to solve the type, satisfy various constraints of $\eta_{i+1}, \dot{\eta}_{i+1}, \ddot{\eta}_{i+1}$. This method is known as the generalized -α projection method. By solving the differential algebraic equation model of multi-agent formation control, the constraints at all levels can be kept accurately and the formation configuration of multi-agent formation can be controlled accurately.

## 4. Numerical Example

The above method is verified and analyzed by taking the replenishment formation action of ocean surface ships as an example.

Equations of motion of a single ship in fixed coordinate system [21]:

$$\begin{cases} \dot{\eta} = R(\psi)v \\ M\dot{v} + C(v)v + D(v)v + g(\eta) = \tau \end{cases} \tag{12}$$

In the formula above, $\eta = \begin{bmatrix} x \\ y \\ \psi \end{bmatrix}$ is the position vector in the ground solid coordinate system, $(x, y)$ is the position of the ship on the sea, $\psi$ is the course Angle. $v = \begin{bmatrix} u \\ v \\ r \end{bmatrix}$ is the velocity vector of the hull in fixed coordinates. The matrices $M, C$ and $D$ represent denote inertia, Coriolis plus centrifugal and damping, $g$ is a vector of generalized forces. $R(\psi)$ represents the rotation matrix of transformation between two coordinate systems,

$$R(\psi) = \begin{bmatrix} cos\,\psi & -sin\,\psi & 0 \\ sin\,\psi & cos\,\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}, \tag{13}$$

It can be simplified as follows:

$$M_\eta \ddot{\eta} + (C_\eta + D_\eta)\dot{\eta} + g_\eta = \tau_\eta \tag{14}$$

In the formula above, $M_\eta(\eta) = R(\psi)MR(\psi)^T$, $C_\eta = R(\psi)\left[C(v) - MR(\psi)^T\dot{R}(\psi)\right]R(\psi)^T$, $D_\eta = R(\psi)D(v)R(\psi)^T$, $g_\eta = R(\psi)g(\eta)$, $\tau_\eta = R(\psi)\tau$ 。

For the formation problem of three ships [11]: There are three surface ships to form the desired formation to complete the actual work activities such as fitting by a specific formation configuration. The value can be:

$$M_i = \begin{bmatrix} 180.3 & 0 & 0 \\ 0 & 2.436 & 1.3095 \\ 0 & 1.3095 & 172.2 \end{bmatrix} \times 10^6 \tag{15}$$



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$$C_i = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 8.61 \\ 0 & -8.61 & 0 \end{bmatrix} \times 10^6$$

$$D_i = \begin{bmatrix} -2.393 \times 10^{-3}|u_i| & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 3.883 \times 10^{-9} & 0 & 0 \\ 0 & 0.2181 & -3.434 \\ 0 & 3.706 & 26.54 \end{bmatrix} \times 10^6$$

The differential algebraic equation model of three-ship formation control and the equation obtained by Baumgarte stabilization method are:

$$\begin{cases} \begin{bmatrix} M_{\eta_1} & 0 & 0 \\ 0 & M_{\eta_2} & 0 \\ 0 & 0 & M_{\eta_3} \end{bmatrix} \ddot{\eta} + \begin{bmatrix} C_{\eta_1} + D_{\eta_1} & 0 & 0 \\ 0 & C_{\eta_2} + D_{\eta_2} & 0 \\ 0 & 0 & C_{\eta_3} + D_{\eta_3} \end{bmatrix} \dot{\eta} = \begin{bmatrix} \tau_{\eta_1} \\ \tau_{\eta_2} \\ \tau_{\eta_3} \end{bmatrix}, \\ \ddot{C}(\eta) = W(\eta)\ddot{\eta} + \dot{W}(\eta)\dot{\eta} = 0. \end{cases} \tag{16}$$

$$\begin{cases} \begin{bmatrix} M_{\eta_1} & 0 & 0 \\ 0 & M_{\eta_2} & 0 \\ 0 & 0 & M_{\eta_3} \end{bmatrix} \ddot{\eta} + \begin{bmatrix} C_{\eta_1} + D_{\eta_1} & 0 & 0 \\ 0 & C_{\eta_2} + D_{\eta_2} & 0 \\ 0 & 0 & C_{\eta_3} + D_{\eta_3} \end{bmatrix} \dot{\eta} = \begin{bmatrix} \tau_{\eta_1} \\ \tau_{\eta_2} \\ \tau_{\eta_3} \end{bmatrix}, \\ \ddot{C}(\eta) = W(\eta)\ddot{\eta} + \dot{W}(\eta)\dot{\eta} = -K_d\dot{C}(\eta) - K_pC(\eta). \end{cases} \tag{17}$$

The above equation simplifies to:

$$\begin{cases} M(\eta, t)\ddot{\eta} + n(\dot{\eta}, \eta, t) = \tau, \\ \ddot{C}(\eta) = W(\eta)\ddot{\eta} + \dot{W}(\eta)\dot{\eta} = 0. \end{cases} \tag{18}$$

$$\begin{cases} M(\eta, t)\ddot{\eta} + n(\dot{\eta}, \eta, t) = \tau, \\ \ddot{C}(\eta) = W(\eta)\ddot{\eta} + \dot{W}(\eta)\dot{\eta} = -K_d\dot{C}(\eta) - K_pC(\eta). \end{cases} \tag{19}$$

Under the definition of position variable $\eta = [\eta_1^T, \eta_2^T, \eta_3^T]^T$, $\eta_i, i = 1,2,3$, the target formation can be represented by constraint function $C(\eta) = 0$. Suppose that three ships form a triangular formation, and let $r_{ij}$ represent the expected distance between ship $i$ and ship $j$. At the same time, the trajectory of one of the ships is constrained by trigonometric function. The constraint function is written as:

$$C(\eta) = \begin{bmatrix} C_r(\eta) \\ C_p(\eta) \end{bmatrix} = 0 \tag{20}$$

The above equation simplifies to:

$$C_r(\eta) = \begin{bmatrix} (x_1 - x_2)^2 + (y_1 - y_2)^2 - {r_{12}}^2 \\ (x_2 - x_3)^2 + (y_2 - y_3)^2 - {r_{23}}^2 \\ (x_3 - x_1)^2 + (y_3 - y_1)^2 - {r_{31}}^2 \end{bmatrix}, \; C_p(\eta) = \begin{bmatrix} t \\ Asin(wt) \\ \text{atan}\,(Awcos(wt)) \end{bmatrix}. \tag{21}$$

Using RK method to solve the model can be obtained:



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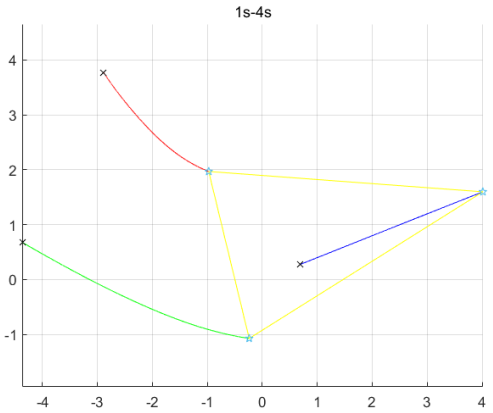


**Fig 1. Location diagram of three ships of Baumgarte stabilization method**

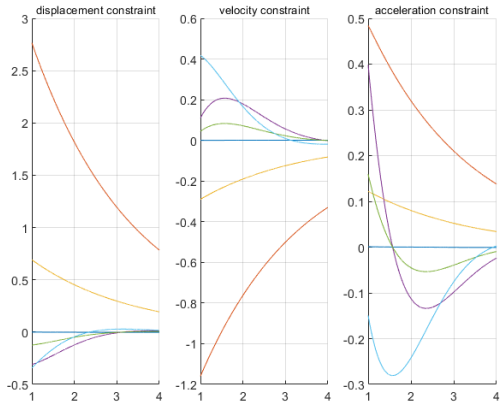


**Fig 2. Three-ship constraint diagram of Baumgarte stabilization method**

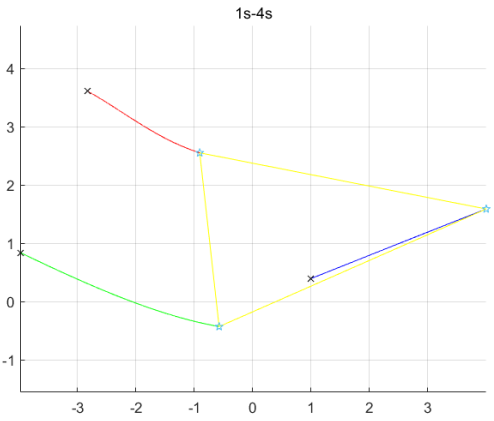


**Fig 3. Location diagram of the third ship of the generalized α projection method**

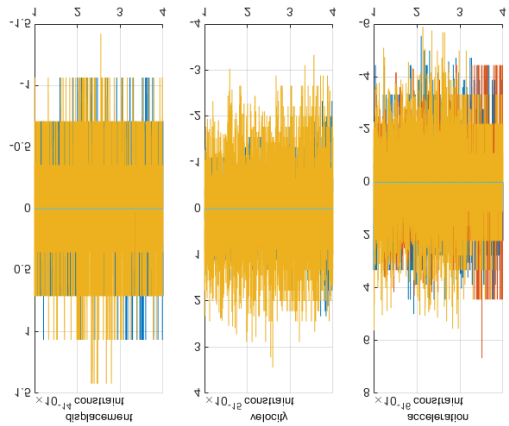

**Fig 4. Three-ship constraint diagram of generalized -α projection method**

The generalized -α projection method can be used to obtain the model:

It can be seen from the above image that the generalized -α projection method is more accurate to the constraints, and the accuracy of displacement constraints is higher. The three levels of constraints are in the order of $10^{-14}$, that is, the control is more accurate than the Baumgarte stability method.

The following is a long time simulation control:

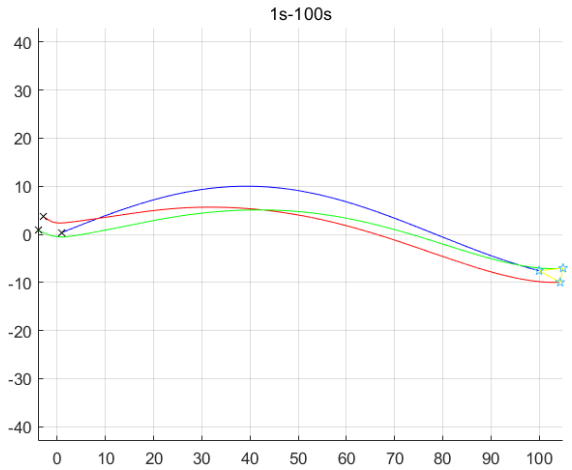


**Fig 5. Location diagram of the third ship of the generalized -α projection method**



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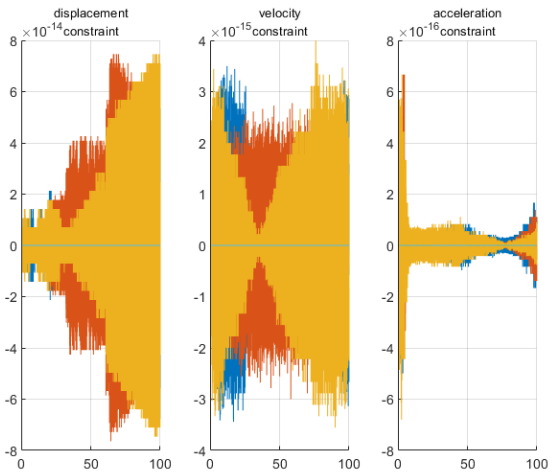


**Fig 6. Three-ship constraint diagram of generalized -α projection method**

It can be seen that the constraints of the generalized -α projection method are still accurate, and the third-level constraints are still in the order of $10^{-14}$, so the long-term control can be kept accurate.

When the selection of method parameters of Baumgarte stable method changes, the following situation can be obtained:

From the above image, it can be found that the constraint keeping condition of Baumgarte stabilization method has changed significantly, the accuracy of acceleration level constraint keeping is low, and the magnitude has changed significantly from before. That is, the parameters of the Baumgarte stable method have a great influence on the results, and there is no efficient method for the selection of parameters. For the generalized -α projection method, there is no need to select the parameters precisely, and it can keep the displacement constraint more precisely and control it more precisely.

## 5. Conclusion

This article for multi-agent formation of the Lagrange system configurations of precise control problem, was proposed based on differential algebraic equation of the generalized - alpha projection control method, the method of generalized alpha projection method in traditional multi-body system dynamics, the superiority of constraint accurately maintain formation of multi-agent system to realize accurate control. Numerical experiments are carried out on ship, a common multi-agent system, and the results are compared with the existing Baumgarte stabilization method. The numerical results of the above examples show that, compared with the Baumgarte stable method, the generalized -α projection method can accurately maintain the formation configuration without selecting the method parameters accurately, and the accuracy of the formation constraint can reach the order of $10^{-14}$, and the control accuracy will not decrease significantly for a long time.

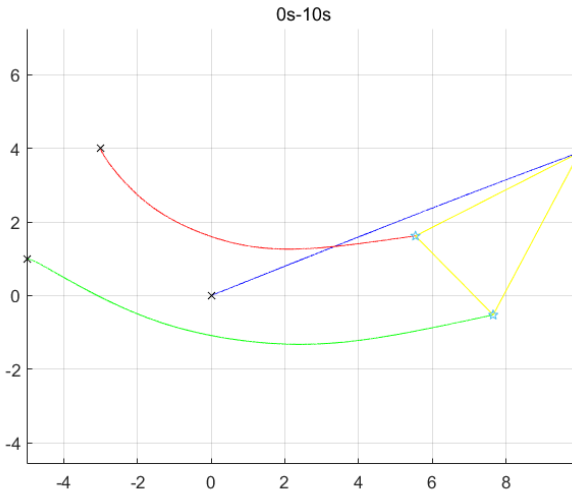


**Fig 7. Baumgarte stabilization method three ship location diagram**

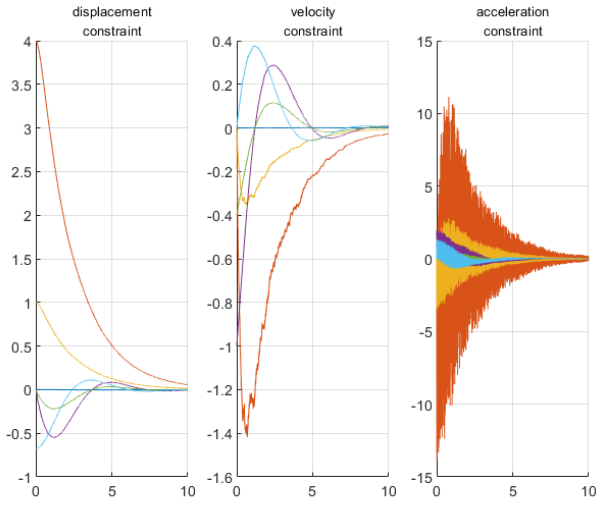


**Fig 8. Three-ship constraint diagram of Baumgarte stabilization method**



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