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Active vibration control of space flexible beam based on multi-agent consensus theory

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Abstract: **Vibration suppression of solar panels, space-borne antennas, grasping manipulators and other components in most spacecraft can be reduced to dynamic control of flexible beam structures.Based on Euler-Bernoulli beam theory and Hamilton principle, the dynamic model of flexible beams under vibration is established, this paper presents a simple and feasible scheme for active vibration suppression of flexible beams based on multi-agent consensus theory.The state variables of two adjacent agents are used to construct the state feedback control law. The control input is realized by piezoelectric ceramic and other intelligent materials. The vibration equation is discretized by differential quadrature method and numerical simulation is carried out.The numerical results show that the control method can effectively suppress the vibration of the flexible beam, reduce its vibration amplitude, and has good anti-interference ability.**

Keywords: Flexible beam structure, Vibration suppression, Multi-agent, Consensus algorithm

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1.Introduction

With the development and utilization of space resources, aerospace technology has developed rapidly in recent years. In aerospace engineering, flexible space structure, as a kind of engineering component with strong function, high flexibility and mature design technology[1], has been increasingly applied to the construction of spacecraft. Flexible beam structure, as its representative, frequently appears in various key parts of spacecraft, such as expandable Antenna antenna[2,3], space manipulator[4,5] and solar panel[6]. However, the interference of external environment and attitude maneuvers inevitably lead to the vibration of these flexible accessories during the mission execution[7]. at the same time, due to the small damping in space, the vibration of the flexible accessories is difficult to decay naturally. If the vibration phenomenon is not controlled, the stability of spacecraft attitude control system and the control accuracy of orbit operation will be greatly affected [8], which will cause a serious threat to the safety of spacecraft.

At present, vibration control methods for flexible spacecraft structures are mainly divided into active control[9-11] and passive control[12,13]. Although the passive control method has the advantages of easy implementation, low cost and simple structure, it basically has no coping ability to deal with sudden environmental changes and lacks flexibility in control. However, with the rise and development of smart materials such as piezoelectric ceramics, the active control based on piezoelectric ceramics with adaptive control ability has been widely used in recent years and has achieved rich results.Zhu et al.[14] used the optimized filtering x minimum mean square deviation algorithm to achieve vibration suppression of piezoelectric flexible cantilever beams, and demonstrated the robustness of the algorithm in different noise environments.Hong Ming and Chen Feng[15] realized active vibration control of piezoelectric laminated cantilever beam by considering the piezoelectric laminated element containing four nodes and using the optimal linear quadratic regulator.Xu et al.[16] designed a decentralized adaptive fuzzy control algorithm to suppress the vibration of the frame composed of the space membrane structure and the piezoelectric actuator.As the basis for the coordinated control of multi-agent systems, the consistency algorithm is widely used in cluster behavior and sensor networks, such as spacecraft attitude control, formation control and other fields[17-19].However, as a typical sensor-actuator network, there are still few studies on the consistency algorithm for the active control problem of flexible beams based on smart materials.

Based on multi-agent consensus theory, this paper puts forward a new kind of active control scheme, First, under the premise of not changing the working mechanism of the space flexible beam structure, according to the Hamilton principle, to establish a simple applicable to Euler-Bernoulli beam mathematical model, and then the stability of beam structure and vibration state as a new virtual leader - follower multi-agent systems, The multi-agent consensus algorithm is used to design the vibration reduction control law, and the numerical simulation experiment is performed to test the control effect of the proposed scheme.

2. Dynamic modeling of flexible beam structure

Space-borne antennas are mostly composed of lightweight and flexible elastic materials, which are typical flexible beam structures in spacecraft design. In order to accurately describe the vibration performance of space-borne antennas, At present, the more commonly used beam mechanical models include the Euler-Bernoulli beam model and Timoshenko Beam model[20].Compared with the Timoshenko beam model, the Euler-Bernoulli beam model ignores the moment of inertia of the beam section around the neutral axis and the shear deformation of the beam, and is mostly suitable for beams whose

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height is much larger than their length.This characteristic exists in many spacecraft beam structures, so the Euler-Bernoulli beam model used in this paper is suitable for most scenarios.FIG. 1 shows a simple schematic diagram of the flexible beam structure.

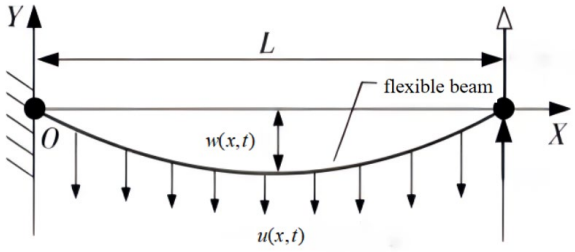

Fig 1. Schematic diagram of the flexible beam type structure

Where, $w(x,t)$ is the lateral vibration offset of the flexible beam, $u(x,t)$ is the discrete control input, and L is the length of the flexible beam.Based on the Euler-Bernoulli beam assumption, the Hamiltonian principle suggests that the energy change of the system is zero during any period of time. It can be obtained by analyzing the energy of the flexible beam:

$$\int_{t_0}^{t_1}\left[\delta E_k-(\delta E_p+\delta W)\right]dt=0 \tag{1}$$

Where, E_k is the kinetic energy of the system, E_p is the potential energy of the system, and W is the total work of external forces on the system.Since the Euler-Bernoulli beam has the rigid property of non-elongation in axial direction, the kinetic energy E_k and E_p potential energy can be expressed as:

$$E_k=\frac{1}{2}\rho A\int_0^L(\frac{\partial w}{\partial t})^2dx \tag{2}$$

$$E_p=\frac{1}{2}EI\int_0^L(\frac{\partial^2 w}{\partial x^2})^2dx \tag{3}$$

Where ρ is the linear density of the flexible beam and A is its cross-sectional area. EI Is the bending stiffness of the flexible beam, and the total work of the external force acting on the system can be expressed as:

$$W=-\int_0^L u(x,t)w(x,t)dx \tag{4}$$

By substituting the above three equations into the Hamiltonian equation, and using the integrating by parts and the variational principle[21], the dynamic equation of the flexible beam structure with equal cross section can be obtained as follows:

$$\frac{\partial^2}{\partial x^2}[EI\frac{\partial^2 W(x,t)}{\partial x^2}]+\rho A\frac{\partial^2 W(x,t)}{\partial t^2}=u(x,t),x\in(0,L),t>0 \tag{5}$$

Among them, the common boundary combination conditions of flexible beam structures are as follows:

Boundary conditions of clamped-clamped beam ($C-C$) :

$$w(x_0,t)=0,\frac{dw(x_0,t)}{dx}=0,w(x_L,t)=0,\frac{dw(x_L,t)}{dx}=0 \tag{6}$$

Boundary conditions of clamped - simply supported beam ($C-S$) :

$$w(x_0,t)=0,\frac{dw(x_0,t)}{dx}=0,w(x_L,t)=0,\frac{dw^2(x_L,t)}{dx^2}=0 \tag{7}$$

Boundary conditions of simply supported - simply supported beam ($S-S$) :

$$w(x_0,t)=0,\frac{dw^2(x_0,t)}{dx^2}=0,w(x_L,t)=0,\frac{dw^2(x_L,t)}{dx^2}=0 \tag{8}$$

Boundary conditions of clamped - free beam ($C-F$) :

$$w(x_0,t)=0,\frac{dw(x_0,t)}{dx}=0,\frac{dw^2(x_L,t)}{dx^2}=0,\frac{dw^3(x_L,t)}{dx^3}=0 \tag{9}$$

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3. Active suppression strategy based on multi-agent consensus theory

Multi-agent system as an important branch of distributed artificial intelligence, the consensus theory has always been one of the hot spot in the field, the basic idea is that every agent according to their own state information and the neighbor state information, through the design consistency algorithm to all agents of state variables converge in a limited time [22], namely the control objectives are:

$$\lim_{t\to\infty}\left\|x_i - x_j\right\| = 0,\ i, j, \in 1,2\cdots N \tag{10}$$

Where N is the number of agents in the system, $x_i(t)$ and $x_j(t)$ are respectively the states of the i and j agents in the system at time t.

The multi-agent system is often analyzed by the knowledge of graph theory, $G=(V,E,A)$ denoted as a weighted directed graph, whose node set is $V=(1,2,\cdots,N)$, $E\subseteq V\times V$ is its edge set, and a weighted directed edge from j to i represents the information that the i agent receives directly from the j agent, $A=(a_{ij})$ represents the weight matrix of the edges, and embodies the communication association among the agents.

The beam vibration model obtained in the first part is set as the object to be controlled and regarded as the follower in the multi-agent system, and the static state of the flexible beam structure is set as the virtual leader.This means that the follower in this system changes according to the state of the leader, and virtual leader to accept information from the follower, but its status will not change because of the follower.These two parts form a leader-follower multi-agent system, as shown in Figure. 2:

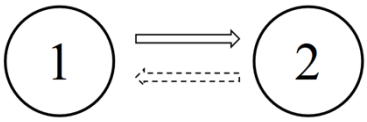

Fig 2. Virtual leader-follower system

The state variables in the system are $\left(w(x,t),\dfrac{\partial w(x,t)}{\partial t}\right)$, where agent 1 is the static state of the flexible beam and agent 2 is the unstable state of the vibration. It is a common and mature method to construct the state feedback control law by using the state variables of two adjacent agents[23]. The following consistent feedback controller is adopted:

$$u_i(x,t)=k\sum_{j=1}^{2}\{a_{ij}(w_j(x,t)-w_i(x,t))+a_{ij}(\frac{\partial w_j(x,t)}{\partial t}-\frac{\partial w_i(x,t)}{\partial t})\},\quad i=1,2 \tag{11}$$

Where, i and j are two adjacent agents, in this system, the corresponding positions of each intelligent sensor under the static state and vibration state of the flexible beam are respectively represented, k is the feedback gain in the controller.

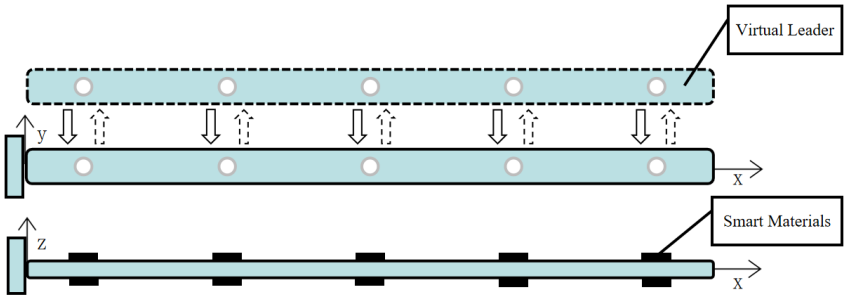

Fig 3. Schematic of active suppression of flexible beams with piezoelectric smart materials

The above consistent control law can be implemented on flexible beam structures by smart materials[24-26] with sensing and actuation functions, such as piezoelectric ceramics and Piezoelectric polymers. Fig 3. shows the active suppression schematic of the flexible beam with piezoelectric smart material based on the above design, piezoelectric material as the actuators and sensors placed in the upper and lower sides of a flexible beam,the above piezoelectric material can detect the vibration state of the flexible beam and the piezoelectric material located below implements the control input from the feedback control law, its working principle is to use are piezoelectric and inverse piezoelectric effect of piezoelectric materials, The electrical signal of the control input is converted into force signal and acted to the specified part of the flexible beam structure, So $u(x,t)$ in the controller can replace the right-hand side of the vibration equation as an external force to suppress the vibration of the

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flexible beam structure.

4. Differential quadrature method

Equation of vibration of fourth order beam is a kind of complex partial differential equation, its analytical solution is difficult to get out. In order to obtain accurate simulation results, this paper used differential quadrature method[27] for numerical solution, its advantage is simple mathematical principle, only a small number of nodes can achieve ideal data accuracy. Especially in the sensor system with limited number of nodes, this method can be used effectively for numerical simulation.The basic idea of differential quadrature method is illustrated by taking unary function $f(x)$ as an example.

Suppose that the function $f(x)$ has an r-order derivative on an interval $[a,b]$ and discretize it by n points. Then according to the basic assumption of differential quadrature method, the value of the function at these n point can be approximated by the linear weighted sum of the function values at this point, namely:

$$\left.\frac{\partial^r f}{\partial x^r}(x)\right|_{x=x_i} = a_{i1}^{(r)} f(x_1) + a_{i2}^{(r)} f(x_2) + \ldots + a_{iN}^{(r)} f(x_N) = \sum_{k=1}^{N} a_{ik}^{(r)} f(x_k) \quad i = 1,2,\ldots,n \tag{12}$$

$a_{ik}^{(r)}$ called the weighted coefficient of the r-order derivative of the function $f(x)$, and the matrix $A^{(r)}$ composed is called the r -order weighted coefficient matrix. When $r=1$, the above equation is expanded into a matrix form:

$$\begin{bmatrix} \frac{d}{dx} f(x_1) \\ \frac{d}{dx} f(x_2) \\ \vdots \\ \frac{d}{dx} f(x_n) \end{bmatrix} = \begin{bmatrix} A_{11}^{(1)} & A_{12}^{(1)} & \cdots & A_{1n}^{(1)} \\ A_{21}^{(1)} & A_{22}^{(1)} & \cdots & A_{2n}^{(1)} \\ \vdots & \vdots & \ddots & \vdots \\ A_{n1}^{(1)} & A_{n2}^{(1)} & \cdots & A_{1n}^{(1)} \end{bmatrix} \begin{bmatrix} f(x_1) \\ f(x_2) \\ \vdots \\ f(x_n) \end{bmatrix} \tag{13}$$

Because

$$\frac{d^r}{dx^r} f(x) = \frac{d}{dx}\left[\frac{d^{r-1}}{dx^{r-1}} f(x)\right] = \frac{d^{r-1}}{dx^{r-1}}\left[\frac{d}{dx} f(x)\right] \tag{14}$$

So the weighting coefficient matrix of each derivative has the following relationship:

$$A^{(r)} = A^{(1)} A^{(r-1)} = A^{(r-1)} A^{(1)} \tag{15}$$

In order to obtain the weight coefficient matrix, different interpolation polynomials can be used to approximate the function $f(x)$, and then the expression of weight coefficients of differential quadrature method can be obtained by taking the derivatives. The following uses Lagrangian triangular interpolation polynomial to illustrate.

$$f(x) \approx \tilde{f}(x) = \sum_{j=1}^{N} f(x_j) l_j(x) \tag{16}$$

Where, $l_j(x) = \prod_{\substack{k=1 \\ k \neq j}}^{N} \frac{\sin(\frac{x - x_k}{2})\pi}{\sin(\frac{x_j - x_k}{2})\pi}$,the approximate representation of the first-order to m -order derivative at any point can be obtained by taking the derivative of both sides of the above equation:

$$\frac{\partial^m f(x_i)}{\partial x^m} \approx \frac{\partial^m \tilde{f}(x_i)}{\partial x^m} = \sum_{j=1}^{N} \frac{\partial^m l_j(x_i)}{\partial x^m} f(x_j), (i = 1,2,\cdots,N) \tag{17}$$

When $m=1$,the weighting coefficient of the first derivative of the function $f(x)$ can be obtained, which is denoted by a_{ij} :

$$a_{ij} = \frac{\pi}{2} \frac{M(x_i)}{M(x_j) \sin\frac{x_i - x_j}{2}\pi}, j \neq i, a_{ii} = -\sum_{j=0, j\neq i}^{N} a_{ij} \tag{18}$$

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Where $M(x_i)=\prod_{\substack{k=0\\k\neq i}}^{N}\sin(\frac{x_i-x_k}{2})\pi$ .Using the fourth-order weight coefficient matrix, the fourth-order spatial derivative in the original partial differential equation can be replaced, and the equation can be transformed into an ordinary differential equation containing only time variables, and then solved by the Runge-Kutta method to obtain the system state variables.

5. Simulation analysis

Taking the flexible beam structure with fixed support at both ends as an example, the following parameters[2] are taken: the linear density of the flexible beam: $\rho=7.8\times10^3 kg/m^2$, the bending stiffness of the flexible beam section: $EI=0.25N/m$, the beam cross section width: $a=3.42mm$, the beam cross section length: $b=30mm$, and the flexible beam length: $l=1m$ .The gain coefficient: $k=6.5$, and the weight a_{ij} of the edge (v_i,v_j) are taken as 1.Given an initial disturbance: $Q(x,0)=(-1)^i*0.1*x^2(x-1)^2, x\in[0,1]$, the feedback control law described above is used for simulation:

Table 1: Comparison of the results of differential quadrature method with different number of nodes

Number of spatial nodes	Running time /s	Boundary constraint error	Control after stabilization Maximum error
5	0.8845	0.04204	0.01084
7	0.9274	0.006963	0.001773
9	0.9794	0.001083	0.0003187
11	-	-	-

Table 1 is the result comparison of the number of spatial nodes when the time step $h=0.001$ is selected and the space node is Chebyshev node.When the space node is set to 5, the maximum error after the control is stable is large, and the result is not ideal. Larger number of nodes increases, the error of the boundary constraint function decreases, and the largest relative error is reduced. But as nodes grow, matrix becomes singular, although can further decrease the time step, or directly to increase the number of node space to avoid, but run time will increase. Therefore, the number of nodes in the following simulation is selected as 9. At this time, the constraint conditions are well maintained and the simulation operation efficiency is good.

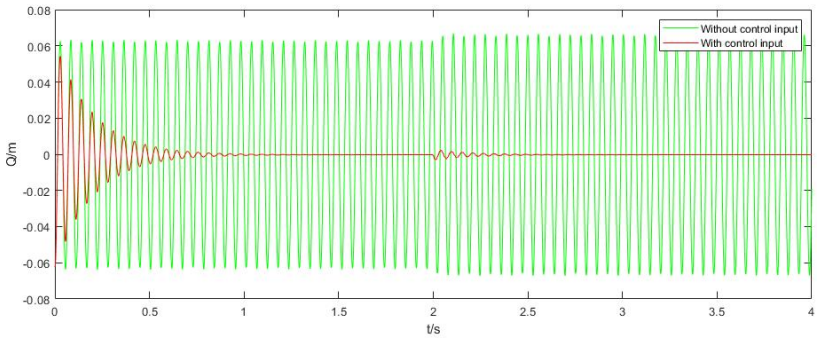

Fig 4. Time history diagram of midpoint displacement of flexible beam

In figure 4, the red line is the time history diagram of the displacement of the midpoint of a flexible beam with nine chebyshev nodes in four seconds, time step $h=0.001$. The green line for midpoint displacement time history without control. From the left side of the image, it can be seen that the vibration phenomenon of the flexible beam structure is significantly weakened in the first 1.5 seconds.After the controller acts for 1.5 seconds, the flexible beam structure is basically in a stable state, and if the system is not controlled, it will always be in a vibrating state.After the system runs for two seconds and reaches the stable state, an external interference force of 200N is applied to it at $t=2$ to simulate the unstable factors in the environment, as shown in the figure 4, under the environmental interference, the suppression effect is obvious, and the controller runs stably.

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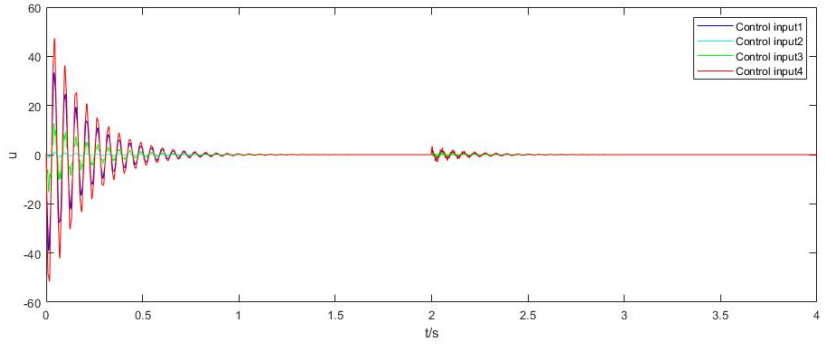

Fig 5. Variation of control input force at each control node

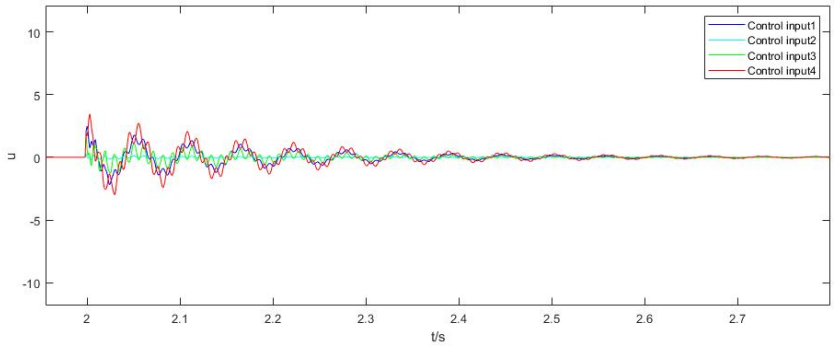

Fig 6. Variation of control input force at each control node after simulated application of environmental interference force (enlarged)

Figure 5 shows the control force variation of each actuator on the flexible beam. Due to the symmetric nine-point simulation, So list the simulation data for the three on the left and the middle actuator.Figure. 6 shows the control input changes of each actuator after the environmental interference force is applied. As can be seen in Figure 5, the control force size and variation range of each actuator are the largest at the midpoint of the most intense vibration, and the control force of each actuator tends to 0 as the system becomes stable.Compared with Figure 5 and Figure 6, the control input of the actuator can operate normally under the influence of the external interference force applied at $t = 2$, and the vibration can still be suppressed.

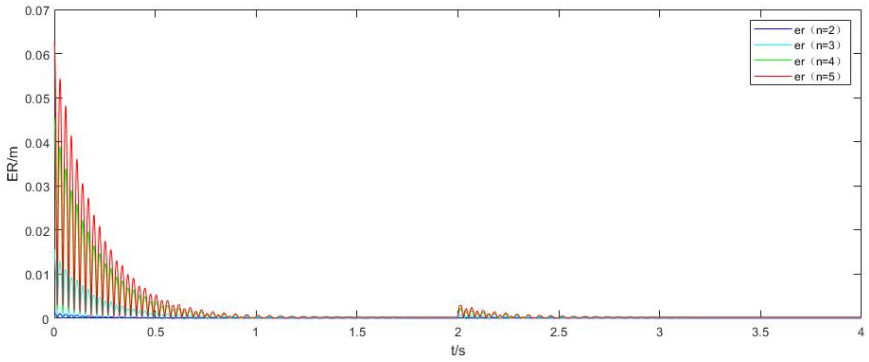

Fig 7. Relative error variation of each control node

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As shown in Figure 6, Let $er = \frac{\sum |Q_1(x,t) - Q_2(x,t)|}{n}, x \in (0, L)$,the average error variation between the vibration suppression process and the stable state of the flexible beam can be obtained. The analysis of the error diagram shows that the error gradually decreases with the increase of time, and the flexible beam structure tends to be stable under this control law, and the control effect is obvious.Moreover, the controller can still suppress the vibration effectively after the interference force is applied.

6. Conclusion

This paper introduces a kind of active vibration control based on multi-agent consensus theory, through the establishment of a virtual leader-follower system, and by using the vibration of a flexible beam and steady-state state variable to construct a feedback control law, the control inputs and condition monitoring are realized by smart materials such as piezoelectric ceramics. And by using the differential quadrature method for numerical simulation.According to the results of numerical examples, a more appropriate number of nodes is given, and used this number of nodes for simulation analysis. The simulation data show that the control method can effectively suppress the vibration of the flexible beam, reduce its vibration amplitude, and has a certain anti-interference ability, has good control performance.The proposed method is simple and feasible, but it can be used to further analyze the spatial distribution of sensors and the communication delay of smart materials, which can bring more ideas and methods to the active control of space flexible beam structures.

Acknowledgments

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