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# Complexiton solutions to the generalized (3+1)-dimensional Shallow Water-like equation

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***Abstract:*** **In this paper, the extended transformed rational function method is used to obtain complexiton solutions of the generalized (3+1)-dimensional Shallow Water-like equation. With the help of computer symbols, we can better understand the properties of the solutions of the equation.**

***Keywords:*** Complexiton solutions; Generalized (3+1)-dimensional Shallow Water-likeequation; Symbolic computation

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## 1. Introduction

Nonlinear evolution equations play a vital important role in the fields of mathematics, physics, optical soliton communication, computer engineering and so on. That is of great significance and value to find the exact solutions of these nonlinear evolution equations. Analytical solutions of nonlinear evolution equations have been researched with a large number of effective and direct methods including Hirota's bilinear method[1,2], the homogeneous balance method[3-5], the tanh-coth method[6,7], the sech-function method[8], the tanh-function method[9,10], the extended tanh-function method[11,12], the Painlevė analysis method[13], exponential function method[14-16], the (G'/G)-expansion method[17], F-expansion method[18], symmetry reductions[19,20],etc. Complexiton solutions[21] belongs to exact solutions newly introduced of nonlinear evolution equations. The key step in the process to obtain complexiton solutions is to get the Hirota's bilinear equations. And the Complexiton solutions are very important, which has important value in both theory and practice.

In this paper, we first obtain complexiton solutions of the generalized (3+1)-dimensional Shallow Water-like equation with the help of the bilinear operator in Ref [22], which introduced rational solutions and lump solutions to the equation. And Grammian solutions have been gotten in Ref[23]. The solution and qualitative a-

nalysis of the nonlinear wave equation are helpful to explain the law of motion change of the system under the nonlinear action.

The structure of this paper is organized as follows. In Section 2, we present the complexiton solutions of the generalized (3+1)-dimensional Shallow Water-like equation and show its phenomena of complexiton with figures. In Section 3, we make a few concluding remarks.

## 2. The generalized (3+1)-dimensional Shallow Water-like equation

The generalized (3+1)-dimensional Shallow Water-like equation as follows

$$u_{xxxy}+3u_{xx}u_y+3u_xu_{xy}-u_{yt}-u_{xz}=0. \tag{2.1}$$

If $z=x$, the generalized (3+1)-dimensional Shallow Water-like equation turns to be the dimensionally reduced equation[22], which also have complexiton solutions.

Under the transformation $u=2(\ln f)_x$, equation (2.1) has the Hirota bilinear form as follows

$$(D_x^3D_y-D_yD_t-D_xD_z)f\cdot f=0, \tag{2.2}$$

where the operators $D_x$, $D_t$, $D_y$ and $D_z$ are original Hirota derivatives. Defined by

$$D_t^mD_x^n(f\cdot g)=(\partial_t-\partial_{t'})^m(\partial_x-\partial_{x'})^n\times f(x,t)g(x',t')\big|_{x'=x,t'=t}. \tag{2.3}$$

The above equation(2.2) gives

$$f_{xxxy}f-f\cdot f_{yt}-f\cdot f_{xz}-f_{xxx}f_y+f_yf_t-3f_{xxy}f_x+f_xf_z+3f_{xx}f_{xy}=0, \tag{2.4}$$

where $f=f(x,y,z,t)$.

In order to search for traveling wave solutions, we determine the solution of equation (2.1) as the following form

$$u^{(\gamma)}(\zeta)=\kappa(\eta)=\frac{p(\eta)}{q(\eta)}=\frac{p_m\eta^m+p_{m-1}\eta^{(m-1)}+\ldots+p_0}{q_m\eta^m+q_{m-1}\eta^{(m-1)}+\ldots+q_0}, \tag{2.5}$$



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where $p(\eta),\ q(\eta)$ are polynomials and $\gamma > 0$ expresses the minimal differential number in ordinary differential equations by traveling wave transformation. For convenience, we choose that

$$p(\eta_1,\eta_2) = A\eta_1 + B\eta_2, q(\eta_1,\eta_2) = 1. \tag{2.6}$$

In other words

$$f = A\eta_1 + B\eta_2, \tag{2.7}$$

where $\eta_1$ and $\eta_2$ are determined by the principle as below

$$\eta_1'' = \frac{d^2\eta_1}{d\zeta_1^2} = -\eta_1, \tag{2.8}$$

$$\eta_2'' = \frac{d^2\eta_2}{d\zeta_2^2} = \eta_2, \tag{2.9}$$

where $\zeta_1 = k_1x + bk_1y + ck_1z + \omega_1t + c_1, \zeta_2 = k_2x + bk_2y + ck_2z + \omega_2t + c_2,\ A,\ B,\ k_1,\ k_2,$ $\omega_1$ and $\omega_2$ will be determined later, $b,\ c,\ c_1$ and $c_2$ are arbitrary constants. From equation (2.8) and equation (2.9), we obtain the relations

$$\begin{aligned}
f_x &= Ak_1\eta_1' + Bk_2\eta_2',\\
f_y &= Abk_1\eta_1' + Bbk_2\eta_2',\\
f_z &= Ack_1\eta_1' + Bck_2\eta_2',\\
f_t &= A\omega_1\eta_1' + B\omega_2\eta_2',\\
f_{yt} &= Ab\omega_1k_1\eta_1'' + Bb\omega_2k_2\eta_2'' = -Ab\omega_1k_1\eta_1 + Bb\omega_2k_2\eta_2,\\
f_{xz} &= Ack_1^2\eta_1'' + Bck_2^2\eta_2'' = -Ack_1^2\eta_1 + Bck_2^2\eta_2,\\
f_{xx} &= Ak_1^2\eta_1'' + Bk_2^2\eta_2'' = -Ak_1^2\eta_1 + Bk_2^2\eta_2,\\
f_{xy} &= Abk_1^2\eta_1'' + Bbk_2^2\eta_2'' = -Abk_1^2\eta_1 + Bbk_2^2\eta_2,\\
f_{xxy} &= Abk_1^3\eta_1^{(3)} + Bbk_2^3\eta_2^{(3)} = -Abk_1^3\eta_1' + Bbk_2^3\eta_2',\\
f_{xxx} &= Ak_1^3\eta_1^{(3)} + Bk_2^3\eta_2^{(3)} = -Ak_1^3\eta_1' + Bk_2^3\eta_2',\\
f_{xxxy} &= Abk_1^4\eta_1^{(4)} + Bbk_2^4\eta_2^{(4)} = Abk_1^4\eta_1 + Bbk_2^4\eta_2.
\end{aligned} \tag{2.10}$$

It is clearly that equations (2.8) and (2.9) respectively admit the following case solutions

$$\eta_1 = \sin\zeta_1 \ \text{ or } \ \eta_1 = \cos\zeta_1, \tag{2.11}$$

$$\eta_2 = \sinh\zeta_2 \ \text{ or } \ \eta_2 = \cosh\zeta_2, \tag{2.12}$$

and these solutions are related to their derivatives. Substituting equation (2.10) into equation (2.4),we get

$$\begin{aligned}
&2B^2ck_2^2 + 8A^2bk_1^4 - 8B^2bk_2^4 + 2A^2ck_1^2 + 2A^2bk_1\omega_1 + 2B^2bk_2\omega_2 + 2AB\eta_1\eta_2bk_1^4\\
&+2AB\eta_1\eta_2bk_2^4 + 2AB\eta_1\eta_2ck_1^2 - 2AB\eta_1\eta_2ck_2^2 + 2AB\eta_1\eta_2bk_1\omega_1 - 2AB\eta_1\eta_2bk_2\omega_2\\
&+8AB\eta_1'\eta_2'bk_1^3k_2 - 8AB\eta_1'\eta_2'bk_1k_2^3 + 2AB\eta_1'\eta_2'bk_1\omega_2 + 2AB\eta_1'\eta_2'bk_2\omega_1\\
&+4AB\eta_1'\eta_2'ck_1k_2 - 12AB\eta_1\eta_2bk_1^2k_2^2 = 0.
\end{aligned} \tag{2.13}$$

Further, we get a series of algebraic equations by sorting out coefficients of $\eta_1\eta_2,\ \eta_1'\eta_2'$ and the constant term and making their coefficients to be zero. Extraordinarily, the following algebraic equations are obtained under the relations



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of $(\eta_1')^2 = 1-\eta_1^2$ and $(\eta_2')^2 = 1+\eta_2^2$.

$$2B^2ck_2^2 + 8A^2bk_1^4 - 8B^2bk_2^4 + 2A^2ck_1^2 + 2A^2bk_1\omega_1 + 2B^2bk_2\omega_2 = 0, \tag{2.14}$$

$$(2bk_1^4 + 2bk_2^4 + 2ck_1^2 - 2ck_2^2 + 2bk_1\omega_1 - 2bk_2\omega_2 - 12bk_1^2k_2^2)AB = 0, \tag{2.15}$$

$$(8bk_1^3k_2 - 8bk_1k_2^3 + 2bk_1\omega_2 + 2bk_2\omega_1 + 4ck_1k_2)AB = 0. \tag{2.16}$$

To solve the above equations (2.14)-(2.16), we get

$$k_2 = k_1, A = B, \tag{2.17}$$

$$c = \frac{-b(2k_1^3 + \omega_2)}{k_1}, \tag{2.18}$$

$$\omega_1 = 4k_1^3 + \omega_2, \tag{2.19}$$

**or**

$$k_2 = -k_1, A = B, \tag{2.20}$$

$$c = \frac{b(-2k_1^3 + \omega_2)}{k_1}, \tag{2.21}$$

$$\omega_1 = 4k_1^3 - \omega_2. \tag{2.22}$$

Substituting equations (2.11), (2.12), with equations (2.17)-(2.22), into equation (2.7), respectively, we get the complexiton solutions of the generalized (3+1)-dimensional Shallow Water-like equation as follows

$$u = 2(\ln f)_x, \tag{2.23}$$

$$f(x,y,z,t) = A\sin[k_1x + (4k_1^3 + \omega_2)t + bk_1y - (2bk_1^3 + b\omega_2)z + c_1]$$
$$\pm A\sinh[k_1x + \omega_2t + bk_1y - (2bk_1^3 + b\omega_2)z + c_2], \tag{2.24}$$

$$f(x,y,z,t) = A\cos[k_1x + (4k_1^3 + \omega_2)t + bk_1y - (2bk_1^3 + b\omega_2)z + c_1]$$
$$\pm A\sinh[k_1x + \omega_2t + bk_1y - (2bk_1^3 + b\omega_2)z + c_2], \tag{2.25}$$

and

$$f(x,y,z,t) = A\sin[k_1x + (4k_1^3 - \omega_2)t + bk_1y + \frac{b(-2k_1^3 + \omega_2)}{k_1}k_1z + c_1]$$
$$\pm A\sinh[-k_1x + \omega_2t - bk_1y - \frac{b(-2k_1^3 + \omega_2)}{k_1}k_1z + c_2], \tag{2.26}$$

$$f(x,y,z,t) = A\cos[k_1x + (4k_1^3 - \omega_2)t + bk_1y + \frac{b(-2k_1^3 + \omega_2)}{k_1}k_1z + c_1]$$
$$\pm A\sinh[-k_1x + \omega_2t - bk_1y - \frac{b(-2k_1^3 + \omega_2)}{k_1}k_1z + c_2], \tag{2.27}$$

where $A$, $k_1$, $\omega_2$, $b$, $c$, $c_1$ and $c_2$ are arbitrary constants.

Substituting equations (2.24) and (2.25) into equation (2.23), two of the complexiton solutions of the generalized (3+1)-dimensional Shallow Water-like equation as below

$$u = (2(A\cos(-k_1x - (4k_1^3 + \omega_2)t - bk_1y + (2bk_1^3 + b\omega_2)z - c_1)k_1$$
$$\pm A\cosh(-k_1x - \omega_2t - bk_1y + (2bk_1^3 + b\omega_2)z - c_2)k_1)) / (-A\sin(-k_1x - (4k_1^3 + \omega_2)t$$
$$-bk_1y + (2bk_1^3 + b\omega_2)z - c_1) \mp A\sinh(-k_1x - \omega_2t - bk_1y + (2bk_1^3 + b\omega_2)z - c_2)), \tag{2.28}$$



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$$u=(2(A\sin(-k_1x-(4k_1^3+\omega_2)t-bk_1y+(2bk_1^3+b\omega_2)z-c_1)k_1 \mp A\cosh(-k_1x-\omega_2t-bk_1y+(2bk_1^3+b\omega_2)z-c_2)k_1))/(-A\cos(-k_1x-(4k_1^3+\omega_2)t -bk_1y+(2bk_1^3+b\omega_2)z-c_1)\pm A\sinh(-k_1x-\omega_2t-bk_1y+(2bk_1^3+b\omega_2)z-c_2)), \quad \textbf{(2.29)}$$

where $A,\ k_1,\ \omega_2,\ b,\ c,\ c_1$ and $c_2$ are arbitrary constants.
In order to further investigate of the solutions, we present graphs as follows.

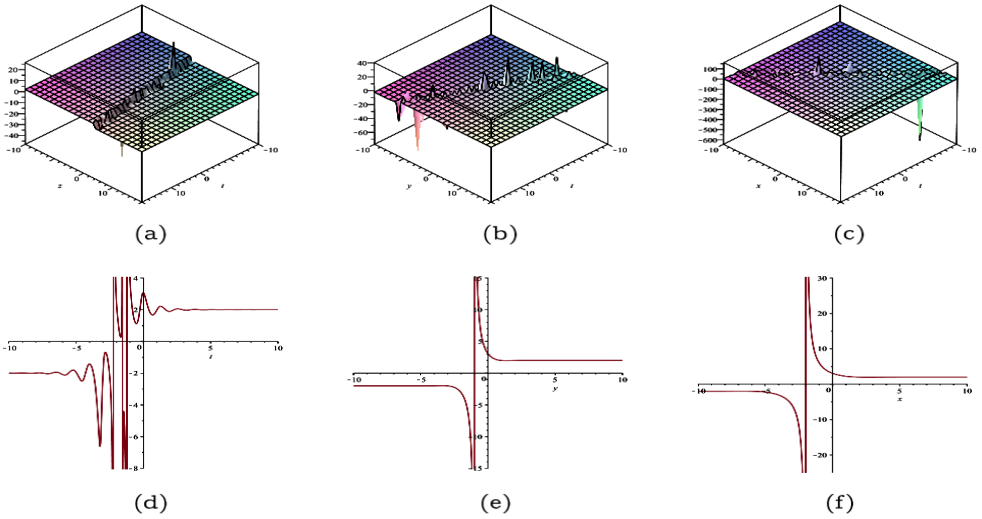


**Figure 1. Propagation of the complexiton solutions forequation (2.28)with parameters:** $k_1=1,\ \omega_2=1,\ c_1=2,\ c_2=2,\ A=2,\ b=2.$ **Perspective view of the solutions with:** $(a)x=0,\ y=0.\ (b)x=0,\ z=0.\ (c)y=0,\ z=0.$ **Wave propagation pattern of the wave along with: (d) The** $t$ **axis. (e) The** $y$ **axis. (f) The** $x$ **axis.**

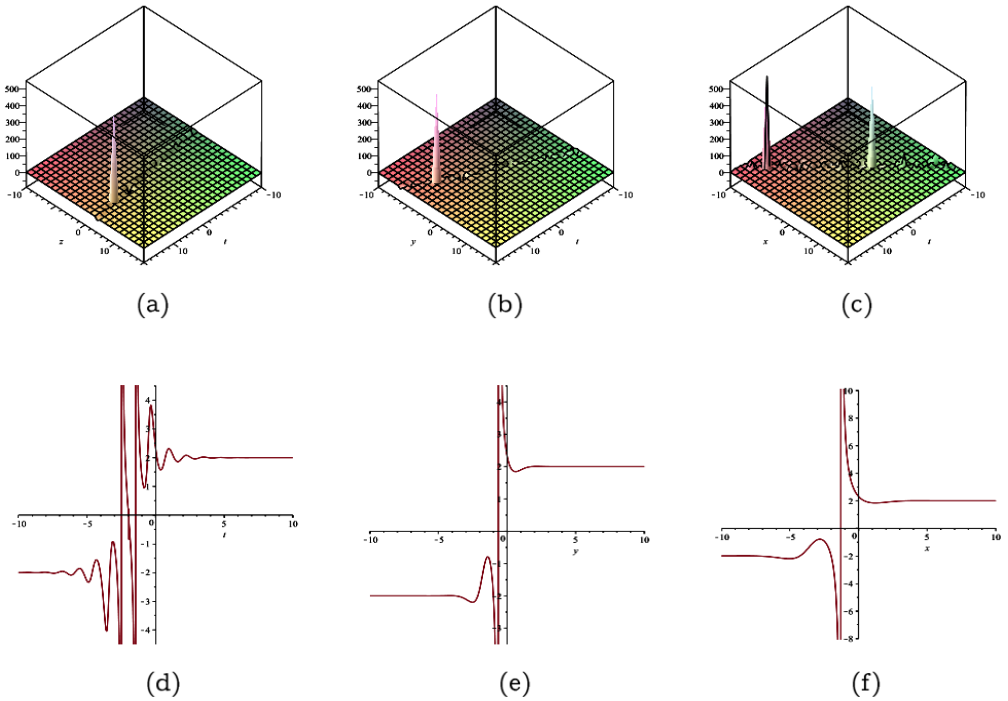


**Figure 2: Propagation of the complexiton solutions for equation(2.29)with parameters:** $k_1=1,\ \omega_2=1,\ c_1=2,\ c_2=2,\ A=2,\ b=2.$ **Perspective view of the solutions with:** $(a)x=0,\ y=0.\ (b)x=0,\ z=0.\ (c)y=0,$ **Wave propagation pattern of the wave along with: (d) The** $t$ **axis. (e) The** $y$ **axis. (f) The** $x$ **axis.**



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## 3. Conclusion

The extended transformed rational function method is more systematic and convenient. Such solutions are more meaningful to study the exact solutions of the generalized (3+1)-dimensional Shallow Water-like equation.

We are convinced that there should exist complexiton solutions to other soliton equations, forinstance, a (3+1)-dimensional nonlinear evolution equation $u_{yt}-u_{xxxy}-3(u_xu_y)_x-3u_{xx}+3u_{zz}=0.$

Under the transformation

$$u=2[\ln f(x,y,z,t)]_x=2\frac{f_x(x,y,z,t)}{f(x,y,z,t)},$$

the (3+1)-dimensional nonlinear evolution equation has the Hirota bilinear form

$$(D_tD_y-D_x^3D_y-3D_x^2+3D_z^2)f\cdot f=0.$$

We obtain the complexiton solutions of the (3+1)-dimensional nonlinear evolution equation

$$u=(2(A\cos(k_1x+bk_1y+ck_1z-\frac{k_1(-2bk_1^2+3c^2-3)t}{b}+c_1)k_1$$

$$\pm A\cosh(k_1x+bk_1y+ck_1z-\frac{k_1(2bk_1^2+3c^2-3)t}{b}+c_2)k_1))/(A\sin(k_1x+bk_1y+ck_1z$$

$$-\frac{k_1(-2bk_1^2+3c^2-3)t}{b}+c_1)\pm A\sinh(k_1x+bk_1y+ck_1z-\frac{k_1(2bk_1^2+3c^2-3)t}{b}+c_2)),$$

$$u=(2(-A\sin(k_1x+bk_1y+ck_1z-\frac{k_1(-2bk_1^2+3c^2-3)t}{b}+c_1)k_1$$

$$\mp A\cosh(k_1x+bk_1y+ck_1z-\frac{k_1(2bk_1^2+3c^2-3)t}{b}+c_2)k_1))/(A\cos(k_1x+bk_1y+ck_1z$$

$$-\frac{k_1(-2bk_1^2+3c^2-3)t}{b}+c_1)\mp A\sinh(k_1x+bk_1y+ck_1z-\frac{k_1(2bk_1^2+3c^2-3)t}{b}+c_2)),$$

where $A$, $k_1$, $b$, $c$, $c_1$ and $c_2$ are arbitrary constants.

In the future, we will be looking for more complexiton solutions of soliton equations. The complexiton solutions would help us to cognize a great diversity of motions for the nonlinear waves described by nonlinear evolution equations.

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