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The Soliton and Double Solution of Nonlinear Klein-Gordon Equation Derived from Heisenberg'S Nonlinear Spinor Equation

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Abstract: **In this paper, we had obtained the soliton and double solution of the nonlinear Klein-gordon equation (3) derived from Heisenberg's nonlinear spinor equation. All the terms in the solutions of our nonlinear field equation have the function structures associated with 2^n-pole moment of energy distribution, which should represent respectively the particles with different spin values.**

Keywords: Unified description of elementary particles; New explanation of wave-particle duality

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1. Introduction

We think the singular solution parts should be some soliton solutions, and the proper solution parts should be Eigen-functions of usual quantum mechanics. Therefore, we can give the new explanation to wave-particle duality.

By using Langevin equation with quantum noise given by us, not only we can give Brownian bridge path integral, but also we can explain the new physical sense of double solution.

2. The Nonlinear Klein-Gordon Equation Derived from Heisenberg's Nonlinear Spinor Equation

The famous Heisenberg's nonlinear spinor equation [1] is

$$\left\{\gamma_\mu \partial_\mu - l_0^2(\overline{\psi}\gamma_\mu\gamma_5\psi)\gamma_\mu\gamma_5\right\}\psi = 0 \quad (1)$$

Let us now apply to (1) the operator

$$\left\{\gamma_\mu \partial_\mu + l_0^2(\overline{\psi}\gamma_\mu\gamma_5\psi)\gamma_\mu\gamma_5\right\},$$

then, by using $r_\mu^2 = 1, r_5^2 = 1,$ we obtain as a consequence of (l):

$$\left\{\partial_\mu^2 - l_0^4(\overline{\psi}\psi)^2\right\}\psi = 0, \quad (2)$$

that is

$$\partial_\mu^2\psi - l_0^4 |\psi|^4 \psi = 0, \quad (3)$$

which is new nonlinear Klein-Gordon equation derived by us.

We will be obtaining the soliton and double solutions of the nonlinear field equation(3).

3. The Soliton and Double Solution of the Nonlinear Klein-Gordon Equation

Using the spherical coodinates, the equation(3)can be written as equation(4).

Using perturbation method, we can solve the nonlinear field equation(4).

We shall assume that ψ can be expanded in powers of ε in the form(4a).

We then substitute(4a)into(4)we obtain(5).

We require this equation to be satisfied for ε small but arbitrary. we must therefore equate the coefficients of successive powers of ε on both sides. thus we obtain equation (6),(7),(7a),(7b).

3.1.The Solutions of the Nonlinear Field Equation (3)

First, we solve the equation(6).The time separation may be made by the substitution

$$\psi_0(r、\theta、\varphi,\ t) = \Phi(r、\theta、\varphi)T(t),$$

Therefore, the equation(6) decomposes into two equations:

$$T'' + K^2C^2T = 0 \quad , \quad (8a)$$

$$\Delta_3\Phi + K^2\Phi = 0 \quad . \quad (8b)$$

The solution of (8a) is

$$T(t) = Ce^{ikct} + De^{-ikct}, (\omega = kc). \quad (9a)$$

And the solutions of(8b)are

$$\begin{cases} \Phi_{kl}(r、\theta、\varphi) = \sqrt{\dfrac{\pi}{2kr}}J_{l+\frac{1}{2}}(kr)Y_L(\theta、\varphi)\ ,\ l=1.\ 2.\ 3,?\ \cdots \\ \Phi_{ko}(r、\theta、\varphi) = \dfrac{1}{r}e^{\pm ikr}, \end{cases}$$

(9b)

$\Phi_{kl}(r、\theta、\varphi)$ are the particular solutions of Helmholtz equation(8b), $Y_L(\theta,\varphi)$ are the spherical harmonic functions, $\sqrt{\frac{\pi}{2kr}}J_{l+\frac{1}{2}}(kr) = j_l(kr)$ are the Bessel functions of the first kind, and the second kind are $\sqrt{\frac{\pi}{2kr}}N_{l+\frac{1}{2}}(kr) = n_l(kr)$, these linear dependence solutions can form Hankel functions:

$h_l^{(1)}(kr)=j_l(kr)+in_l(kr), h_l^{(2)}(kr)=j_l(kr)-in_l(kr),$

they contain such terms as $\frac{1}{r}e^{\pm ikr}$.

The solutions of equation(7)should have the same form as the solutions of the non-homogeneous electromagnetic potential equation. Therefore, we can write the solutions of the nonlinear field equation(7a)as equation(10).

Obviously,

$$l_0^4\psi_0^5=\left\{l_0^4\left(\sqrt{\frac{\pi}{2kr}}J_{l+\frac{1}{2}}(kr')\,\Upsilon_L(\theta',\varphi')e^{-i(kr-wt)}\right)^5\right\} \quad (11)$$

as the potential-energy density concentrating in the small microscopic realm. Therefore, we can define

$$\begin{cases}\int\Omega_0 dV'=p_0,\\ \int x_i\Omega_0 dV'=p_i,\\ \int x_i x_j\Omega_0 dV'=p_{ij},\end{cases} \quad (12)$$

representing respectively the total energy, dipole moment and quadruple moment,……They should be constituted by the potential-energy density Ω_0 of dipoles on spherical shell. We suppose here x_i are the components of The radius vector of Ω_0 .And each particle pair on spherical shell may be considered as the dipole with charge G lying in the potential ψ_0 of external field; the probability density of each particle pair should be $\psi_0\psi_0{}^*.\overline{\psi}_0\overline{\psi}_0{}^*$, thus, the potential-energy density Ω_0 of particle pair system on spherical shell should be G $(\psi_0\psi_0{}^*.\overline{\psi}_0\overline{\psi}_0{}^*)\psi_0\infty\psi_0^5$.

Since ψ_l have the same form as the solution of the non-homogeneous electromagnetic potential equation, therefore, ψ_l can be written as the multiple expansion form equation(13).

$$\psi_l(r、\theta、\varphi,t)=\frac{p_0}{R}-\sum p_i\frac{\partial}{\partial x_i}\frac{1}{R}+\sum p_{ij}\frac{\partial}{\partial x_i}\frac{\partial}{\partial x_j}\frac{1}{R} \quad (13)$$

x_i being the components of the radius vector of field point.

According to the potential produced by the 2^n-pole moment of electric charges, the solution ψ_1 can also be written as equation(14).

Using the similar method, we can also calculate $\psi_2,\psi_3,\ ......,$ unto ψ_k, ,and $\psi_{k+l}=0$ .Therefore, equation(4a) becomes the finite sum, and it must be the solution of equation(3). According to non-linear superposition principle, the solutions of the nonlinear field equation(3)not only contain linear superposition terms, but also must add the interaction term to these solutions.

3.2. The Mathematical Structure and Physical Meaning of the Solutions of the Nonlinear Field Eouation (3)

Comparing(13)and(14),we see that the tensor of order n contained in the coefficient of every term $\frac{1}{r^n}$ should be the 2^n Pole moment of energy distribution. And from(12),we see that the tensor of order n should be constituted by the energy distribution and the components of the radius vector. Therefore, the coefficient of every term $\frac{1}{r^n}$ in solution ψ_1 may be considered as the function structure associated with dipole, quadruple or other multiple distribution.

According to Gerard Petiau's view point[3], these function structure to describe the 2^n-pole moment of energy distribution should represent a series of particles with different spins. 0bviously, $C_1(\theta、\varphi),\ \cdots\cdots$ in equation (14)should be some functions relating to the distribution of energy density-the 2^n-pole moment of energy distribution.

In solutions ψ_0 and ψ_1,the every term of their radial wave functions contains such tech terms as $\frac{1}{r}e^{ik'r}$,and

$$\frac{e^{ik'r}}{r}=\frac{\cos k'r}{r}+i\frac{\sin k'r}{r}. \quad (15)$$

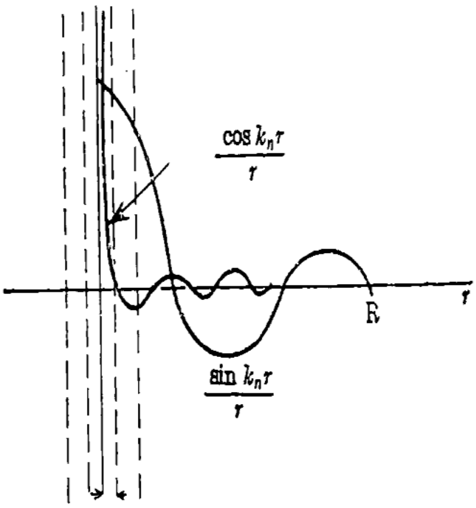

Figure 1. The function $(\frac{e^{ik'r}}{r})$ can be decomposed into Proper and singular parts.

$$\frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2}-\left[\frac{\partial}{r^2\partial r}(r^2\frac{\partial\psi}{\partial r})+\frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi}{\partial\varphi^2}+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta)\frac{\partial\psi}{\partial\theta}\right]=l_0^4\psi^5, \tag{4}$$

$$\psi(r、\theta、\psi,\ t)=\sum_{k=0}^{\infty}\psi_k(r、\theta、\psi,t)\varepsilon^k, k=0,1,2\cdots\cdots, \tag{4a}$$

$$\sum_{k=0}^{\infty}\frac{1}{c^2}\frac{\partial^2\psi_k}{\partial t^2}\varepsilon^k-\left[\sum_{k=0}^{\infty}\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial\psi_k}{\partial r})\varepsilon^k+\sum_{k=0}^{\infty}\frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi_k}{\partial\varphi^2}\varepsilon^k\right.$$
$$\left.+\sum_{k=0}^{\infty}\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta)\frac{\partial\psi_k}{\partial\theta}\varepsilon^k\right]=l_0^4\sum_{k=1}^{\infty}(\sum_{l=0}^{k-1}\psi_l\psi_{k-1-l})\varepsilon^k\psi_0^3,$$
$$\sum_{k=0}^{\infty}\frac{1}{c^2}\frac{\partial^2\psi_k}{\partial t^2}\varepsilon^k-\left[\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial\psi_0}{\partial r})+\sum_{k=1}^{\infty}\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial\psi_k}{\partial r})\varepsilon^k\right.$$
$$+\frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi_0}{\partial\varphi^2}+\sum_{k=1}^{\infty}\frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi_k}{\partial\varphi^2}\varepsilon^k$$
$$\left.+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta)\frac{\partial\psi_0}{\partial\theta}+\sum_{k=1}^{\infty}\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta)\frac{\partial\psi_k}{\partial\theta}\varepsilon^k\right]$$
$$=l_0^4\sum_{k=1}^{\infty}(\sum_{l=0}^{k-1}\psi_l\psi_{k-1-l})\psi_0^3\varepsilon^k. \tag{5}$$

for 0th-order terms:

$$\frac{1}{c^2}\frac{\partial^2\psi_0}{\partial t^2}-\left[\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial\psi_0}{\partial r})+\frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi_0}{\partial\varphi^2}+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta)\frac{\partial\psi_0}{\partial\theta}\right]=0, \tag{6}$$

for Kth-order terms:

$$\frac{1}{c^2}\frac{\partial^{2\psi_k}}{\partial t^2}-\left[\sum_{k=1}^{\infty}\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial\psi_k}{\partial r})+\sum_{k=1}^{\infty}\frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi_k}{\partial\varphi^2}+\sum_{k=1}^{\infty}\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta)\frac{\partial\psi_k}{\partial\theta}\right]=l_0^4\sum_{k=1}^{\infty}(\sum_{l=0}^{k-1}\psi_l\psi_{k-1-l})\psi_0^3, \tag{7}$$

for 1th-order terms:

$$\frac{1}{c^2}\frac{\partial^2\psi_1}{\partial t^2}-\left[\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial\psi_1}{\partial r})+\frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi_1}{\partial\varphi^2}+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta)\frac{\partial\psi_1}{\partial\theta}\right]=l_0^4\psi_0^5, \tag{7a}$$

for 2th-order terms:

$$\frac{1}{c^2}\frac{\partial^2\psi_2}{\partial r^2}-\left[\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial\psi_2}{\partial r})+\frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi_2}{\partial\varphi^2}+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta)\frac{\partial\psi_2}{\partial\theta}\right]=2l_0^4\psi_0^4\psi_1. \tag{7b}$$

$$\psi_1(r、\theta、\varphi,t)=\iiint\frac{1}{|\bar{r}-\bar{r}'|}\{l_0^4\psi_0^5\}dV'$$
$$=\iiint\frac{1}{|\bar{r}-\bar{r}'|}\left\{l_0^4(\sqrt{\frac{\pi}{2kr}}J_{l+\frac{1}{2}}(kr')Y_l(\theta'、\varphi')e^{-iwt'})^5\right\}dV'$$
$$=\iiint\frac{1}{|\bar{r}-\bar{r}'|}\left\{l_0^4(\sqrt{\frac{\pi}{2kr}}J_{l+\frac{1}{2}}(kr')Y_l(\theta'、\varphi')e^{-i(kr-wt)})^5\right\}dV' \tag{10}$$

$$\psi_l(r、\theta、\varphi)=e^{ik'r}\{C_l(\theta、\varphi)\frac{l}{r^{n+1}}+C_2(\theta、\varphi)\frac{k'}{r^n}+\cdots\cdots$$
$$\left.+C_{n+l}(\theta、\varphi)\frac{k'^n}{r}\right\}, \tag{14}$$

$$\psi(\overline{q(\tau)})=\prod_{j=1}^{n+1}(2\sigma_j\sqrt{\pi})\sigma_\tau e^{\frac{im}{\hbar}(n+1)D_Q}\left(\frac{1}{\sqrt{2\pi\sigma_\tau^2}}\right)^{\frac{1}{2}}e^{\frac{-\overline{q(\tau)}^2}{2\sigma_\tau^2}}\left\{e^{\frac{i}{2\hbar}\overline{pq(\tau)}-\frac{i}{\hbar}\overline{E}\tau}\right\}, \tag{18a}$$

The first term $\frac{\cos k'r}{r}$ holds a dominant position in the small spherical region $(r < l_0)$, which is the singular term of solutions, and these singular terms are the solutions confined in the finite space region in any time, obviously, which represent that energy is not disperse. We think that they should be some soliton solutions[5]; and they are also the singular solutions of De Broglie's double solutions[2]. The second term $\frac{\sin k'r}{r}$ holds a dominant position outside r $> l_0$.According to De Broglie's view point, such terms as $\frac{\sin k'r}{r}$ in the solutions ψ_0 and ψ_1 may be considered as the Eigen-functions ψ_n of usual quantum mechanics.

In Heisenberg's nonlinear spinor equation(1), the Eigen-values of particle mass depend on the nonlinear term which must be a self--energy effect. In our equation, the nonlinear terms $l_0^4\psi_0^5$、$l_0^4\psi_0^4\psi_1$,......, may be considered as the"sources" of some fields. And these"sources" may be considered as some distribution of energy density concentrating in the small microscopic realm V', which should be associated with the $2^n -$ pole moment distribution of energy density.

In the double solution theory[2] of Dirac equation, the particles with Spin value s= $\frac{1}{2}$ should be described by the singular solutions corresponding to dipole form. And for those particles with spin values s $> \frac{1}{2}$,Gerard Petiau found that the singular solutions should be corresponding to multiple forms.

Therefore, the product wave functions consisting of some spinor wave functions should be the wave function of the composite particles. That is, the particles with multiple forms should be consisting of the particles with some dipole forms.

As mentioned above, the nonlinear Klein-Gordon equation (3) may rewrite as the following form

$$\partial_\mu^2\psi_1 = l_0^4\psi_0^5 = l_0^4(\psi A_\mu\psi^*)(\overline{\psi}A_\mu\overline{\psi}^*).(\psi_u\psi_u^*)(\psi_s\psi_s^*)(\psi_d\psi_d^*) \quad (3a)$$

The right side of equation (3) may be considered as the invariant of field sources, which is equal to (density of gluon sea quarks)×(density of valence quarks);and ψ_1 should be the wave functions with different spin values. Disturbing the quark-pairs in sea, the mesons and hadrons can be formed.

4. Langevin Equation With Quantum Noise

We had given the Langevin equation with quantum noise[6]

$$\frac{dr(t)}{dt} - \upsilon(r(t),t) = \frac{P_Q(t)}{m}, \quad (16)$$

Where the momentum P_Q (t) plays the role of quantum noise, and the solution of this equation is Brownian bridge. We had given the corresponding Brownian bridge path integral[6]

$$< q''t'' | q't' >= \int g_{q''|q'}[r(t)]\wp r(t) e^{\frac{i}{\hbar}s[r(t)]}, \quad (17)$$

go through a lot of calculations, we obtain the solution of the modulated plane wave of a free particle[6] equation(18a). and the amplitude modulation factor is also Gaussian function, when t→ o, which is $\delta -$functional wave packet, which should diffuse with time t, when $t \to \infty$, this $\delta -$functional wave packet diffuse into the plane wave solution, which is the proper solution in De Broglie' s double solutions

$$\psi = a\exp\frac{i}{\hbar}\varphi, \varphi = E_Q t - P_Q r \quad ; (18b)$$

and the singular solution in De Broglie's double solutions is written as

$$u = f(x, y, z, t)\, e^{\frac{i}{\hbar}\varphi}, \quad (18c)$$

Where the two solutions have the same phase φ.

5. Some New View Points

5.1. Guidance Formula

We may rewrite (16)as the following form

$$\frac{dr(t)}{dt} - \upsilon(r(t),t) = \frac{P_Q(t)}{m} = \frac{grad\varphi}{m} = V_f. \quad (19)$$

Where we also derive the guidance formula, obviously, the velocity V_f should be velocity fluctuation of a free particle deviating its classical trajectory, E_Q and P_Q in (18b)are the stochastic energy and stochastic momentum of a free particle, which are generated by the quantum fluctuation in vacuum.

5.2. Quantum Potential

We may rewrite De Broglie's quantum potential as following form

$$Q = (\frac{\hbar^2}{2m})\frac{\Box^2\psi}{\psi}, (D_Q = \frac{\hbar}{2M}), \quad (20a)$$

Where D_Q is quantum-diffusional coefficient [6] ,thus Q should be the quantum-diffusional potential. Inserting (18b)into (20),we obtain

$$Q = \frac{P_Q^2}{2m} = E_Q, \quad (20b)$$

Which shows that the stochastic quantum potential Q suffered by a free particle must be equal to the stochastic kinetic energy E_Q of this particle, under the action of the stochastic quantum potential Q or the quantum noise P_Q, a free particle must have probability distribution.

5.3. Quantum Force

We calculate the differential quotient of equation (19), we obtain the new stochastic motion equation of a free particle

$$m\frac{d^2r(t)}{dt^2} - m\dot{\upsilon}(\mathrm{r}(t),t) = ma_f = -\nabla Q, Q = \frac{p_Q^2}{2m}, \quad (21)$$

Where a_f is the fluctuation acceleration deviating classical trajectory, equation (21)shows that the De Broglie's quantum force $(-\nabla Q)$ is just the stochastic fluctuation force exerting on a free particle, and which is just exerted by virtual particles in vacuum.

6. Conclusion

We come to the conclusion: Each term of the solutions of our nonlinear field equation(3) should represent respectively the different particles, and these particles with different spin values should be described by the different function structures associated with the different multiple forms, in each term of the solutions of our nonlinear field equation(3),we think the singular solution parts should be some soliton solutions, which have the function structure forms of the point pole, dipole quadrupole, and other multiples.

Obviously,the proper solution parts of our equation should be the Eigen-functions of usual quantum mechanics. In a word, according to our nonlinear field equation(3), we not only can give the unified description of elementary particles, but also can give the new explanation to wave-particle duality.

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