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Mechanical Model and Analysis of Casing Dislocation Form on Oil-Water Well

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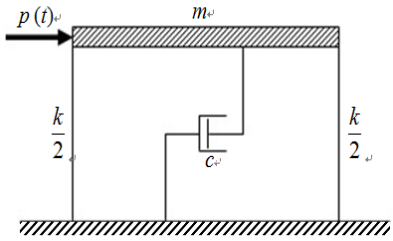
Abstract: **As the service life of the oil-water wells are prolonged, because of the effects of engineering, geology and other factors, the production casings are inevitable to be deformed, damaged even severely dislocated. If these wells do not get reasonable treatments in time, not only it would influence their own normal production, but also affect the production of the whole of area. Nowadays there is no effective means of detection for the form of the casing dislocation. We can't accurately determine the down holes condition when the casing is dislocated. So it is difficult to repair the dislocated casing wells. By establishing mechanical model of casing dislocation form, based on the geologic conditions and statistics of casing damage in the region, according to the conditions of each well such as the parameters of formation, injection-production relationship, the lateral loads of the casing etc, then we use some relative formulae with corresponding parameters to calculate and analyze to predict exactly the situations of casing damage wells including the type of casing damage, the axial and radial displacement of the dislocated casing etc. This method is significant and effective for enhancing the rate of repairing the damage wells and ensuring stable production of the key blocks in Oil field.**



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1. Introduction

After Oil-Water Wells are put into production, Owing to the effects of engineering and geologic factors, including casing material, the quality of cementing, well location, the adjustment of injection-production plan, formation heterogeneity, fault activity etc, the damages of casings are inevitable, even the casing is severely dislocated sometimes [1]. Because the space of underground casing is limited and we can't see the condition of casing dislocation, nowadays the main ways in the process of repairing the damage well, such as lead impression, experiential judgment, probing with special tools aren't effective enough and take a long time [2]. Mechanical model and analysis of casing dislocation form will give effective evidences and judgments for the research on the form of the down hole casing deformation and work over operation. It is significant for enhancing the rate of repairing the damage wells and ensuring stable production of the key blocks in Oil field. Stable production of the key blocks in Oil field.

2. Fundamental assumption of mechanical model

As we know, the stress load that the layer hits on the casing is very huge, so the displacement of the layer is almost equivalent to the displacement of the casing [3-4]. Therefore, we can assume that the constitutive relation of casing material is the ideal rigid-plastic model:

⑴The casing is put vertically in layers, Its material is ideal rigid-plastic model.

⑵ Ignore the influence of radial deformation of thecasing, axial curve and the torque around axis.

⑶Assume that the surfaces of shale and layers are no thickness, which can deliver the tangential force and normal force, cause tangential displacement and normal displacement; Its rigidity will go down with the incensement of relative displacement and injection pressure.

⑷ The restriction of boundary only restrains the vertical displacement and doesn't restrain the horizontal displacement; Top edge, left edge and right edge are free boundaries.

⑸Use units such as quadrilateral etc to simulate the rock, the stress in the centre of the unit stands for its stress state.

3. The condition of judging the casing dislocation

The main reason for the down hole casing dislocation is the powerful shear stress; Casing dislocation will also come out when large volumes of shale is immersed by water, or the faults with high pressure slip [5-6].From the Perspective of the casing load capacity:

$$\tau = \frac{F}{A} \tag{1}$$

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A—Cross-sectional area of the casing;

F—Stress on the casing.

$$A=\frac{\pi}{4}(d_W^2-d_N^2) \quad (2)$$

$$F=d_W hp \quad (3)$$

Take formula (2), get (3) into formula (1), get:

$$\tau=\frac{4d_W hp}{\pi(d_W^2-d_N^2)} \quad (4)$$

So the condition that the casing is dislocated by shear stress is:

$$\tau=\frac{4d_W hp}{\pi(d_W^2-d_N^2)}\geq[\tau]_{xu} \quad (5)$$

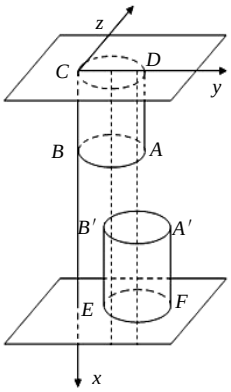

Figure 1. Casing Dislocation Form

4. The model of calculating the transverse displacement of the dislocated casing

After the casing is dislocated, the transverse displacement of the lower-part casing is the result of layer deformation and partial layer slip, so calculating the transverse displacement of the dislocated casing includes two following part.

4.1. The transverse displacement of the casing with the deformation of layers

Because of the existence of injection-production wells system, changing the balance of the original formation pressure will result in the alteration and redistribution of stratum stress field [7-9].The layer is squeezed to deform in the horizontal direction, and it is stretched obviously in the vertical direction. The whole of formation will thrust upward, as shown in figure 2 [10].

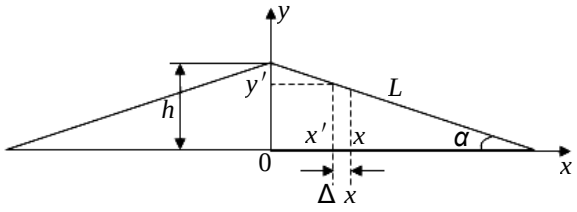

Figure 2. Simplified Schematic Stratigraphic Lift

The transverse and vertical displacements of the casing with the deformation of layer are:

$$\Delta x=|x-x'|=\left|x-\int_0^x\cos\alpha\left[1-\gamma\frac{p(x,t)}{E}\right]dx\right| \quad (6)$$

$$\Delta y=|y'-0|=\left|h-\int_0^x\sin\alpha\left[1-\gamma\frac{p(x,t)}{E}\right]dx\right| \quad (7)$$

In the formula:

Assume that the layer whose original coordinate is (*x*, 0) thrusts by formation pressure which is *p* (*x*, *t*).

γ —Poisson's Ratio;

E—Elastic Modulus;

α —Formation Dip.

By calculating the ratio of Δx and Δy, the probability of the casing deformation can be analyzed. The bigger the ratio of Δx and Δy is, the easier casing deforms. If the casing deforms in the layer with coordinate (*x*,0), the transverse displacement of the casing with the deformation of layers is:

$$\Delta x_1=\Delta x=\left|x-\int_0^x\cos\alpha\left[1-\gamma\frac{p(x,t)}{E}\right]dx\right| \quad (8)$$

4.2. The transverse displacement of the casing with the slip of layers

When the formation pressure alters, differential pressure will come out among partial blocks; Chose the partial blocks as the study object, consider them as SDOF systems under the dynamic loads. When the object with mass is acted by dynamic loads *p*(*t*) which alter with the time going, the oscillation is called forced oscillation. As shown in figure 3, a SDOF system, the mass of object is *m*, the lateral stiffness of the system is k, Damping coefficient is *c*, acted by dynamic loads which is *p*(*t*) [11]. According to Lagrange D' Alembert principle, the dynamic equilibrium equation of the object with mass is:

$$my''+cy'+ky=p(t) \quad (9)$$

Or:

$$y''+2\xi\omega y'+\omega^2 y=\frac{p(t)}{m} \quad (10)$$

$\omega=\sqrt{k/m}$ -circus frequency of the free oscillation with no Damping in the SDOF system;

$\xi=c/2m\omega$ —Damping Ratio.

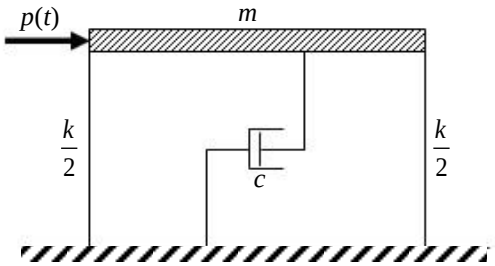

Figure 3. Single Degree of Freedom System under Dynamic Load

Formula (10) is the differential equation of forced oscillation in the SDOF system, which is a 2-order linear inhomogeneous differential equation with constant coefficients, and could be got by algorithm calculation. The transverse displacement of the casing

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with the slip of layers is approximately equivalent to the transverse displacement of the layer, the formula is:

$$\Delta x_2 = \cos\alpha - y(t) = \cos\alpha \cdot \begin{bmatrix} Ae^{-\xi\omega}\sin(\omega_d t+\varphi) \\ +\frac{1}{m\omega_d}\int_0^t p(\tau)e^{-\xi(\omega-\tau)} \\ \times\sin\omega_d(t-\tau)d\tau \end{bmatrix} \quad (11)$$

By all of above-mentioned analyses, we can get the total transverse displacement:

$$\Delta x = \Delta x_1 + \Delta x_2 \quad (12)$$

5. The model of calculating the axial displacement of the dislocated casing

The main reasons for the axial displacement of the casing may be grouped under four heads:

First, the casing was stretched by force loads before it is dislocated, the length of upper-part and lower-part casing shrinks after the casing was dislocated; Secondly, because the casing dislocation usually appears in high pressure layers, the length of the casings after dislocation will shrink due to differential pressure; Thirdly, after the casing is dislocated, the lower-part casing would generate axial displacement due to the deformation of layers; Finally, when the layers slip and there are dips in the layers, the casing will go down with layers.

5.1. The length of the casing shrinking under the axial stress

After the casing run in hole, as a result of own gravity, buoyancy, radial stress and tangential stress, the length will alter [12]. Assume the total length of casing is *H*, the length of lower-part casing after it is dislocated is *h*, and the length of casing under tensile loads after it is dislocated is *H-h*.

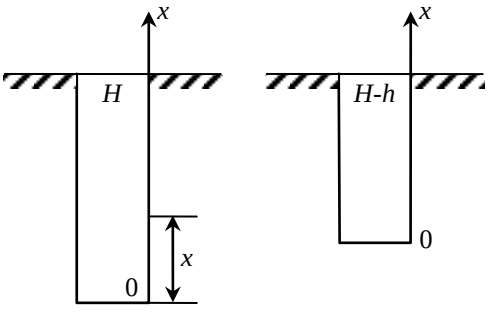

Figure 4. Casing of Tension and Load Section before and after Dislocation Coordinate Diagram

The casing deformation because of axial stress, radial stress and tangential stress is:

$$\varepsilon_t = E^{-1}\left[(\sigma_x - \sigma_c) - \gamma(\sigma_r + \sigma_t)\right] \quad (13)$$

σ_x—Gravity of the casing;

σ_c—Crushing stress due to the buoyancy;

σ_r—Radial stress;

σ_t—Tangential stress;

ε_t—Axial strain of the casing;

γ —Poisson's Ratio.

By means of derivation, the tensile deformation amounts of the casing before dislocation (*L=H*) and the casing after dislocation (*L=H-h*) are:

$$e_{t1} = \frac{H^2 g}{2E}\left[\rho_s - 2\rho_f(1-\gamma)\right] \quad (14)$$

$$e_{t2} = \frac{(H-h)^2 g}{2E}\left[\rho_s - 2\rho_f(1-\gamma)\right] \quad (15)$$

ρ_s—Density of the casing;

ρ_f—Density of the fluid in wellbore.

Therefore, the set-back amount of the length of dislocated casing due to tensile loads under the gravity is:

$$\Delta h_1 = e_{t1} - e_{t2} = \frac{gh}{E}\left(H - \frac{h}{2}\right)\left[\rho_s - 2\rho_f(1-\gamma)\right] \quad (16)$$

Table 1 Casing Cement Ring and Stratum Calculation Parameters

Name	Elastic modulus (104MPa)	Poisson's Ratio	Density (kg•mm-3)
Casing	20.7	0.28	7.28×10-6
Cement ring	4.01	0.23	2.30×10-6
Stratum	1.00	0.25	2.30×10-6

5.2. The set-back amount of the dislocated casing length under differential pressure

Because original tensile loads disappear after the casing is dislocated, the lower-part casing turns to be under pressure differential between layers. So the stretching amount of the lower-part casing will reduce, and the length of lower-part casing will also reduce under pressure [13-14]. The set-back amount of the lower-part casing length under pressure is:

$$\Delta h_2 = \frac{\Delta p_1}{E}h \quad (17)$$

Δp_1—Differential pressure between shale and layer with low pressure.

5.3. The axial displacement of the casing with the deformation of layers

To layers with small dig, the variance of its thickness is able to be described by the theory formula that American scholars apply in the laboratory experiments [15]:

$$\Delta H = \frac{C_f \Delta P H}{1+\phi} \quad (18)$$

H—Effective thickness of layers;

φ —Porosity of layers;

ΔP—Variance of formation pressure;

C_f—Coefficient of volume compressibility of formation.

Assuming that the stretching amount of the casing is equivalent to the variance of layers' thickness,

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according to the theory of Material mechanics, when $\sigma > [\sigma]$ the casing will be dislocated. After the casing is dislocated, the lower-part casing will lose original tensile loads on it, so under this circumstance the set-back amount of the lower-part casing is:

$$\Delta h_3 = \Delta H \tag{19}$$

5.4. The axial displacement of the casing with the slip of layers

Layers slipping can result in the dislocation of casing, because of the existence of dips in the layers; according to geometry we can get the downward displacement of the casing with layer slipping:

$$\Delta h_4 = \sin\alpha \cdot y(t) = \sin\alpha \cdot \begin{bmatrix} Ae^{-\xi\omega}\sin(\omega_d t+\varphi) \\ +\dfrac{1}{m\omega_d}\int_0^t p(\tau)e^{-\xi\omega(t-\tau)} \\ \times \sin\omega_d(t-\tau)d\tau \end{bmatrix} \tag{20}$$

By analyzing above formula, model of the totally axial displacement of casing dislocation is:

$$\Delta h = \Delta h_1 + \Delta h_2 + \Delta h_3 + \Delta h_4$$

Table 2 The Basic Parameters Of Stratum Data Sheet

Parameters	Values
Permeability(μm^2)	0.5
Original formation pressure(MPa)	10
Formation dip(^{o})	2
Damping Ratio	0.05
Time(a)	600
Structure coefficientβ	1.54
Porosity	0.22
Compressibility (1/MPa)	0.00138
Rock mass(kg)	1×10^6
Freqency(Hz)	500
Lifting differential Pressure (MPa)	2
Structure coefficient ψ	0.52

6. The case about calculation and analysis of the model

According to the reality of the casing damage wells in tertiary oil recovery regions of Daqing Oilfield, based on the mechanical model of casing dislocation, we take seven wells in tertiary oil recovery regions as an example.

By analyzing the condition of injection and production in the region, according to statistical regularity curves of casing damage in this area, we get the preliminary judgment that the depth of casing damage is 820-870m. The average thickness of each sub-layer is 2m, the thickness of oil casing is 7.72mm, and then we get the type of casing damage of each well by calculating. According to the conditions of each well such as the parameters of formation, injection and production relationship etc, considering the lateral loads of each well due to the factors such as shale creep, layer slip etc, we can use above-mentioned formulae with corresponding parameters to calculate and analyze.

Table 3 Calculation and Analysis of Dislocation Form Based on Mechanical Model

Well	Predicted Type	Calculation Displacement (mm)	Practical Displacement (mm)
1	deform	/	/
2	dislocate	25	27
3	dislocate	21	20
4	deform	/	/
5	dislocate	20	18
6	dislocate	22	21
7	deform	/	/

Table 3 show us that the judgment of casing damage type is accurate, the calculation of the depth and displacement of casing damage is little different to the reality. In this way we can instruct effectively the next work over operation, it's very meaningful for giving the corresponding measures in the well site.

7. Conclusions

By the study on mechanical model of casing dislocation form, we can use it to judge effectively whether the casing damage wells are dislocated, and use it to analyze and calculate the displacement of the dislocated casing, it is very meaningful and instructional for making the reasonable repairing plan and using special tools. With the constant development of Oil field in the world, work over operations for the damage wells will become more and more important; the number of dislocated casing wells will increase rapidly every year. So it is necessary to apply and modify the mechanical model of casing dislocation form, which can give the powerful help and instruction for the repairing of the damage oil-water well.

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