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Comparison of Finite Difference and Finite Volume Method Basd on MAC for Numerical Simulation of Driven Cavity Flow

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Abstract

In this paper, finite difference method and finite volume method are used to simulate the two-dimensional lid-driven cavity flow based on MAC with staggered grid. Second-order central difference format are applied to discretize the convection and diffusion items respectively. Simulation results are compared by adopting different Reynolds number (100,400,1000) and grid numbers (20×20、40×40、80×80). The two simulation results show that MAC has high calculation precision to solve the problem of driven cavity flow.

1. Introduction

The original state of the driven cavity flow, in a rectangular domain, is assumed to be two-dimensional, static and incompressible. The top-lid of the cavity flow is moving at a positive, constant velocity u_{top} in the positive x−direction

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while the other walls of the cavity keep still. The height and length of the rectangular domain are H. Fluid density is ρ .The dynamic viscosity of the fluid is μ [1].

The purpose of this paper is to determine the dimensionless velocity U in x-direction in the place of $x = \frac{H}{2}$, the dimensionless velocity V in y-direction in the place of $y = \frac{H}{2}$,the mean values of $|U|$ and $|V|$,separately; and draw the velocity distribution curve while the flow is on steady state in the cavity for Re=1000.

2. Mathematical Model

By considering H , ρ , u_{top} dimensionless metric, which were equal to 1,the Navier-Stokes equations were simplified into the dimensionless equations[2].

The dimensionless pressure-Poisson equation can be solved by the TDMA [3].

$$\frac{\partial^2 P}{\partial X^2} + \frac{\partial^2 P}{\partial Y^2} = 2\left(\frac{\partial U}{\partial X}\frac{\partial V}{\partial Y} - \frac{\partial U}{\partial Y}\frac{\partial V}{\partial X}\right). \quad (1)$$

2.1 The finite diferrence method

2.1.1 The description of the grids and conditions[4]

The dimensionless governing equations were discretized with 80×80 uniform-staggered mesh grids. P_{ij} was in the central of the grid. The numbers of U, V deviating from P_{ij} , were the same as the number of P .The dimensionless velocity U was staggered in half a grid with P_{ij} in x-direction and the dimensionless velocity V was staggered in half a grid with P_{ij} in y-direction. The grid generation is described in detail as follows.

a) For pressure P: in x-direction, the distance between interior points is ΔX and the boundary point is $\frac{\Delta X}{2}$ from the interior point, including 83 points from 0 (the left boundary point) to 82 (the right boundary point); in y-direction, the distance between interior points is ΔY and the boundary point is $\frac{\Delta Y}{2}$ from the interior point, including 83 points from0 (the bottom boundary point) to 82 (the top boundary point).

b)For velocity U:In x-direction, the distance between points is ΔX , including 82 points from 0 (the left boundary point) to 81 (the right boundary point);in y-direction, the distance between interior points is ΔY and the boundary point is $\frac{\Delta Y}{2}$ from the interior point, including 83 points from0 (the bottom boundary point) to 82 (the top boundary point).

c)For velocity V: in x-direction, the distance between interior points is ΔX and the boundary point is $\frac{\Delta X}{2}$ from the interior point, including 83 points from0 (the left boundary point) to 82 (the right boundary point); in y-direction, the distance between points is ΔY , including 82 points from 0 (the bottom boundary point) to 81 (the top boundary point);

2.1.2 The process of the discretization based on MAC

The central difference format was used to separately discretize the convective term and the diffusion term of the Navier-Stokes equations. The velocities term were discretized in explicit format and the pressure gradient term adopted the implicit format. The time step

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is $\Delta\tau$, the steps of x, y are separately considered as ΔX , ΔY .The velocity in x-direction was discretized as follows.

The discretization of interior points:

$$\frac{\partial U}{\partial \tau} = \frac{U_{i,j}^{n+1} - U_{i,j}^{n}}{\Delta \tau},$$

$$U\frac{\partial U}{\partial X} = U_{i,j}^{n}\frac{U_{i,j}^{n+1} - U_{i,j}^{n}}{2\Delta X},$$

$$V\frac{\partial U}{\partial Y} = \frac{1}{4}(V_{i,j}^{n} + V_{i+1,j}^{n} + V_{i,j-1}^{n} + V_{i+1,j-1}^{n})\frac{U_{i,j+1}^{n} - U_{i,j-1}^{n}}{2\Delta Y}$$

$$-\frac{\partial P}{\partial X} = -\frac{P_{i+1,j}^{n+1} - P_{i,j}^{n+1}}{\Delta X},$$

$$\frac{\partial^2 U}{\partial X^2} = \frac{U_{i+1,j}^{n} - 2U_{i,j}^{n} + U_{i-1,j}^{n}}{\Delta X^2},$$

$$\frac{\partial^2 U}{\partial Y^2} = \frac{U_{i,j+1}^{n} - 2U_{i,j}^{n} + U_{i,j-1}^{n}}{\Delta Y^2}.$$

The interior points next to the boundary were specially handled, setting the points next to the top boundary as example[5]. Taylor expansion of $U_{i,j+1}$ and $U_{i,j-1}$ were conducted at $U_{i,j}$.

$$\left.\frac{\partial U}{\partial Y}\right|_{i,j} = \frac{4U_{i,j+1}^{n} - 3U_{i,j}^{n} - U_{i,j-1}^{n}}{3\Delta Y} + O(\Delta Y^2), \quad (2)$$

$$\left.\frac{\partial^2 U}{\partial Y^2}\right|_{i,j} = \frac{8U_{i,j+1}^{n} - 12U_{i,j}^{n} + 4U_{i,j-1}^{n}}{3\Delta Y^2} + O(\Delta Y), \quad (3)$$

U can be converted using (2)and(3) to the following form:

$$U_{i,j}^{n+1} = <U_{i,j}^{n}> + \Delta\tau\frac{P_{i,j}^{n+1} - P_{i+1,j}^{n+1}}{\Delta X}. \quad (4)$$

$V_{i,j}^{n+1}$ can be obtained with the same theory.

$$V_{i,j}^{n+1} = <V_{i,j}^{n}> + \Delta\tau\frac{P_{i,j}^{n+1} - P_{i,j+1}^{n+1}}{\Delta Y}. \quad (5)$$

2.2 The finite volume method

The dimensionless governing equations were discretized with 80×80 uniform-staggered mesh grids using the same theory as the finite difference method.

We used the control-volume integration method based on MAC to solve this problem. The central difference format was used to separately discretize the convective term and the diffusion term. The velocities term were discretized in explicit format and the pressure gradient term adopted the implicit format. The time step is $\Delta\tau$, the steps of x, y are separately considered as ΔX , ΔY .

For interior points:

$$\int_s^n\int_w^e\int_t^{t+\Delta t}\frac{\partial U}{\partial\tau}d\tau dX dY = [U_p^{n+1} - U_p^{n}]\Delta X\Delta Y = (U_{i,j}^{n+1} - U_{i,j}^{n})\Delta X\Delta Y$$

$$\int_t^{t+\Delta t}\int_s^n\int_w^e\frac{\partial(UU)}{\partial X}dX dY d\tau = [\frac{(U_{i,j}^{n} + U_{i+1,j}^{n})^2}{4} - \frac{(U_{i,j}^{n} + U_{i-1,j}^{n})^2}{4}]\Delta Y\Delta\tau$$

$$\int_t^{t+\Delta t}\int_w^e\int_s^n\frac{\partial(VU)}{\partial Y}dY dX d\tau = [\frac{(V_{i,j}^{n} + V_{i+1,j}^{n})(U_{i,j}^{n} + U_{i,j+1}^{n})}{4} - \frac{(V_{i,j-1}^{n} + V_{i+1,j-1}^{n})(U_{i,j}^{n} + U_{i,j-1}^{n})}{4}]\Delta X\Delta\tau$$

$$\int_t^{t+\Delta t}\int_s^n\int_w^e\frac{1}{Re}\frac{\partial}{\partial X}(\frac{\partial U}{\partial X})dX dY d\tau = \frac{1}{Re}[\frac{U_{i+1,j}^{n} - U_{i,j}^{n}}{(\delta X)_e} - \frac{U_{i,j}^{n} - U_{i-1,j}^{n}}{(\delta X)_w}]\Delta Y\Delta\tau$$

$$\int_t^{t+\Delta t}\int_w^e\int_s^n\frac{1}{Re}\frac{\partial}{\partial Y}(\frac{\partial U}{\partial Y})dY dX d\tau = \frac{1}{Re}[\frac{(U_{i,j+1}^{n} - U_{i,j}^{n})}{(\delta Y)_n} - \frac{(U_{i,j}^{n} - U_{i,j-1}^{n})}{(\delta Y)_s}]\Delta X\Delta\tau,$$

The pressure term

$$\int_t^{t+\Delta t}\int_s^n\int_w^e\frac{\partial P}{\partial X}dX\,dY\,d\tau = (P_{i+1,j}^{n+1} - P_{i,j}^{n+1})\Delta Y\Delta\tau.$$

The diffusion term of the top and bottom boundary points

$$\frac{1}{Re}(\frac{\partial U}{\partial Y})_n = \frac{1}{Re}\frac{U_n - U_p}{(\frac{\Delta Y}{2})_n} = \frac{1}{Re}\frac{U_{i,81} - U_p}{(\frac{\Delta Y}{2})_n}$$

$$\frac{1}{Re}(\frac{dU}{dY})_s = \frac{1}{Re}\frac{U_p - U_s}{(\frac{\Delta Y}{2})} = \frac{1}{Re}\frac{U_p - U_{i,0}}{(\frac{\Delta Y}{2})}.$$

Then the interior points' velocity U can be converted into the following format

$$U_{i,j}^{n+1} = <U_{i,j}>^{n} + \Delta\tau\frac{P_{i,j}^{n+1} - P_{i+1,j}^{n+1}}{\Delta X}. \quad (6)$$

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The velocity V can be derived in the same theory

$$V_{i,j}^{n+1} = < V_{i,j} >^{n} + \Delta\tau \frac{(P_{i,j}^{n+1} - P_{i,j+1}^{n+1})}{\Delta Y} \quad . \qquad (7)$$

3. Solution

The discretization control equation and its diffusion coefficient expression were shown as follows [6].

$$a_P P_{i.j}^{n+1} = a_E P_{i+1.j}^{n+1} + a_W P_{i-1.j}^{n+1} + a_N P_{i.j+1}^{n+1} + a_S P_{i.j-1}^{n+1} + b \quad , \qquad (8)$$

Where

$$a_E = a_W = \frac{\Delta\tau}{\Delta X^2} ,$$

$$a_N = a_S = \frac{\Delta\tau}{\Delta Y^2} ,$$

$$b = \frac{\langle U_{i-1,j} \rangle^n}{\Delta X} - \frac{\langle U_{i,j} \rangle^n}{\Delta X} + \frac{\langle V_{i,j-1} \rangle^n}{\Delta Y} - \frac{\langle V_{i,j} \rangle^n}{\Delta Y} .$$

The interior points next to the top boundary:

$$a_N = 0, \frac{\langle V_{i,j} \rangle^n}{\Delta Y} = \frac{V_{i,j}^n}{\Delta Y} .$$

The interior points next to the bottom boundary:

$$a_S = 0, \frac{\langle V_{i,j-1} \rangle^n}{\Delta Y} = \frac{V_{i,j-1}^n}{\Delta Y} .$$

The interior points next to the left boundary:

$$a_E = 0, \frac{\langle U_{i-1,j} \rangle^n}{\Delta X} = \frac{U_{i-1,j}^n}{\Delta X} .$$

The interior points next to the right boundary:

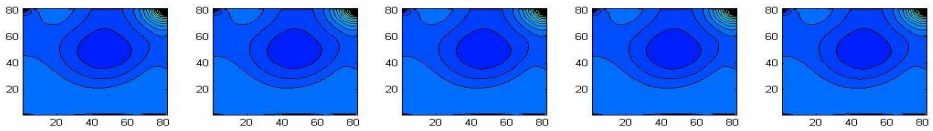
$$a_W = 0, \frac{\langle U_{i,j} \rangle^n}{\Delta X} = \frac{U_{i,j}^n}{\Delta X} .$$

4. Conclusion

According to the results presented in Table 1and figure.1 to figure.3, it's shown that MAC has high calculation precision to solve the problem of driven cavity flow. To accelerate computation speed, 20×20 grid systems can be adopted since results change little in the grid systems of 20×20,40×40 and 80×80.The time of calculation can be shortened by choosing a large time step with little effect on the precision. However, divergence occurred when time step is larger than t=0.02s.

Table 1 iterative times of CACULATION (RE=1000, PRECISION=10^{-7})

Grid numbers	10×10	20×20	40×40	80×80	**Grid numbers**	80×80	80×80	80×80	80×80
t	0.001	0.001	0.001	0.001	**t**	0.0005	0.001	0.002	0.005
Iterative times (P)	178085	284516	471512	708796	**Iterative times (P)**	767452	708796	640421	537223
Iterative times (UV)	51142	47085	38042	36499	**Iterative times (UV)**	65017	36499	20926	10430



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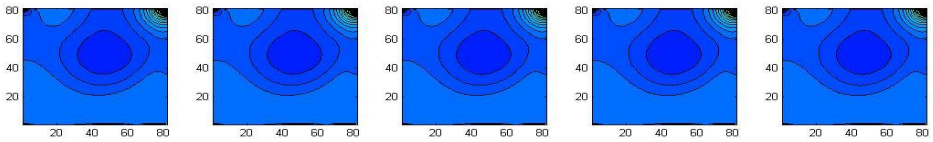

Fig. 1 The velocity distribution of the steady flow while N=80; Re=400; t=0.0005, 0.001, 0.002, 0.005, 0.01 (The upper pictures are results of the finite volume method and the lower pictures are obtained by the finite difference method)

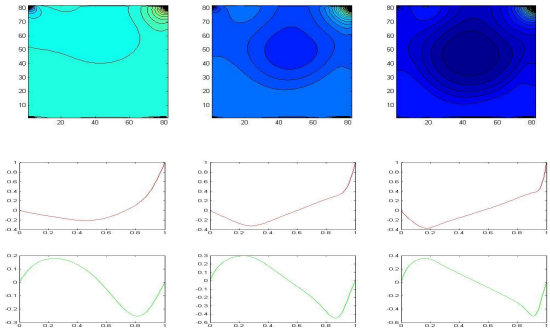

Fig. 2 The velocity distributionof the steady flow determined by the finite volume method while N=80; Re=100,400,1000

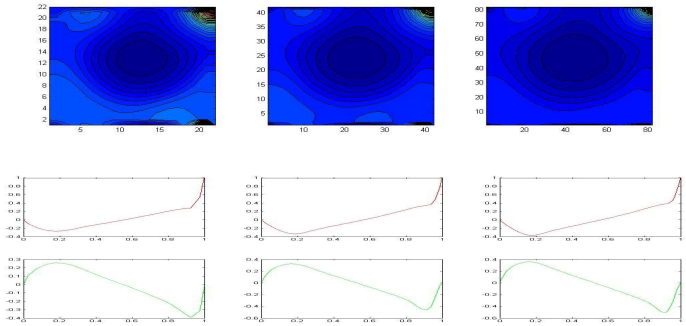

Fig. 3 The velocity distribution while the flow is on steady state by the finite volume method while N=20,40,80; Re=1000

References

[1] Y.H. Robert, R.H. Wang, Computer & Fluids, 2 (2003) 337-352.

[2] W.Q. Tao, Xi'an University of Communications Press, 301:327(in Chinese).

[3] S. Abdallah, Journal of computational physics, 31（1987）182-192.

[4] M. Hortmann, M. Perić, G.J. Scheuerer, International Journal for Numerical Methods in Fluids, 6（1990）189-207.

[5] I. Demirdžić, M. Perić, International Journal for Numerical Methods in Fluids, 9（1990）771-790.

[6] A. Brandt, Springer Berlin/Heidelberg, 12（1973）82-89.

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