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Synchronization Criteria of Impulsive Lurie Systems with Time-delay

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Abstract

This paper is mainly concerned with the issues of synchronization criteria of impulsive Lurie systems with time-delay. Based on stability theory of impulsive differential equation and new comparison theory of impulsive differential system, some simple yet less conservative criteria ensuring impulsive synchronization of the Lurie systems are derived. A numerical example is given to illustrate the effectiveness of the method.

1. Introduction

During the last decade, there has been increasing interest in the subject of chaotic synchronization [1-8]. Synchronization has opened the way to investigate an engineering application of chaos such as chaotic communications. Recently, impulsive synchronization has been widely investigated and many effective methods have been presented. In many practical examples of impulsive control systems, three typical examples are the insect population control system whose state variables are the number of insects and their natural enemies, a chemical reactor system with the quantities of different chemicals server as states variables, and a financial system with two state variables of the amount of money in the market and the saving rates of a central bank. However, the

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existence of time-delay is one of the main factors that may cause the failure of the intended synchronization result.

In this paper, based on stability theory of impulsive differential equation and new comparison theory of impulsive differential system, we study the chaos impulsive synchronization of two coupled Lurie systems using the unidirectional linear error feedback scheme. Some simple generic conditions of chaos impulsive synchronization of two coupled Lurie systems are derived, and to apply the conditions to typical chaotic system-the Chua's oscillator such that chaos impulsive synchronization is achieved.

The organization of this paper is as follows. Some preliminaries relevant to impulsive control theory of dynamical systems are first prepared in Section 2. In Section 3, we introduce some basic definitions and supporting results, where some synchronization criterions of two impulsive Lurie systems are presented. A numerical example is given to show the effectiveness of the method in section 4. Finally, concluding remarks are given in Section 5.

Notation: R^n denotes the n-dimensional Euclidean space, $R^{m\times n}$ is the set of all $m\times n$ real matrices. For symmetric matrices P and Q, the notation (P>Q) means that matrix (P-Q) is positive definite (positive semi-definite). * denotes the elements below the main diagonal of a symmetric block matrix. Denote by $\lambda_{\max}(P), \lambda_{\min}(P)$, respectively, the largest and the smallest eigenvalues of the matrix P.

2. Problem formulation

Consider the Lurie system

$$\dot{x}(t) = Ax(t) + Bx(t-\tau) + D\varphi(C^T x(t)), \quad (1)$$

Where $x \in R^n$ is the state vectors and matrices, $A, B \in R^{n\times n}$, $C, D \in R^{n\times m}$, $\varphi \triangleq (\varphi_1, \varphi_2, \cdots, \varphi_m)^T$, $\varphi_i(\cdot)$ satisfies with $\varphi_i(0) = 0$ and

$$(\varphi_i(\varsigma_i) - k_{1i}\varsigma_i)(\varphi_i(\varsigma_i) - k_{2i}\varsigma_i) \le 0 \ \forall \varsigma_i \text{ for } i = 1,2,\cdots,m \quad (2)$$

where

$$k_{2i} > k_{1i}.$$

Consider a discrete set $\{t_k\}$ of time instants, where $0 < t_1 < t_2 < \cdots < t_k < t_{k+1} < \cdots$, $t_k \to \infty$, as $k \to \infty$. $\{t_k : k = 1,2,\cdots\}$.

also satisfy

$$t_1 = \sup_{1\le k<\infty}\{t_{2k+1} - t_{2k}\} < \infty$$

$$t_2 = \sup_{1\le k<\infty}\{t_{2k} - t_{2k-1}\} < \infty.$$

At time instants t_k jumps in the state variable x are denoted by $\Delta x|_{t=t_k} = x(t_k^+) - x(t_k)$, where

$$x(t_k^+) = \lim_{t\to t_k^+} x(t) \quad k \in \mathrm{N}.$$

Let $I_k \in C[R^n, R^n]$ denote the incremental change of the state at the time t_k, and $I_k(0) = 0$ for all $k \in \mathrm{N}$. Then, we get the following impulsive Lurie system:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bx(t-\tau) + D\varphi(C^T x(t)) & t \ne t_k, \\ \Delta x = I_k(x) & t = t_k, \\ x(t_0^+) = x_0 & t_0 \ge 0,\ k \in \mathrm{N}. \end{cases} \quad (3)$$

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From the unidirectional linear coupling approach, a driven system for (1) is constructed as follows:

$$\dot{y}(t)=Ay(t)+By(t-\tau)+D\varphi(C^T y(t))+H(x-y), \tag{4}$$

where $H=\mathrm{diag}(h_1,h_2,\cdots,h_n)$, with $h_i\in R,\ i=1,2,\cdots,n$, is a feedback matrix to be designed later.

At discrete instances, $t_k,\ k=1,2,\cdots$, the state variable of the driving system are transmitted to the driven system are subjected to jumps at these instants. In this sense, the driving system is given by (1), while the driven system is modeled by the following impulsive equations:

$$\begin{cases}\dot{y}(t)=Ay(t)+By(t-\tau)+D\varphi(C^T y(t))+H(x-y) & t\neq t_k,\\ \Delta y=-I_k(e) & t=t_k,\\ y(t_0^+)=y_0 & t_0\geq 0,\ k\in \mathrm{N}.\end{cases} \tag{5}$$

where $e^T=x^T-y^T=(x_1-y_1,x_2-y_2,\cdots;x_n-y_n)$ is the synchronization error.

Let $\eta(C^T e,y)=\varphi(C^T e+C^T y)-\varphi(C^T y)$,

Then the error system of the impulsive synchronization is given by:

$$\begin{cases}\dot{e}(t)=(A-H)e(t)+Be(t-\tau)+D\eta(C^T e,y) & t\neq t_k,\\ \Delta e=I_k(e) & t=t_k,\\ e(t_0^+)=e_0 & t_0\geq 0,\ k\in \mathrm{N}.\end{cases} \tag{6}$$

Assumes that the nonlinearity $\eta(C^T e,y)$ belong to sector $[K_1,K_2]$,

$$k_{1i}\leq \frac{\eta_i(C_i^T e,y)}{C_i^T e}=\frac{\varphi_i(C_i^T e+C_i^T y)-\varphi_i(C_i^T y)}{C_i^T e}\leq k_{2i}.$$

This implies that

$$(\eta_i(C_i^T e,y)-k_{1i}C_i^T e)(\eta_i(C_i^T e,y)-k_{2i}C_i^T e)\leq 0, \tag{7}$$

so

$$(\eta(C^T e,y)-K_1C^T e)^T(\eta(C^T e,y)-K_2C^T e)\leq 0, \tag{8}$$

where

$K_1=\mathrm{diag}(k_{11},k_{12},\cdots,k_{1m})$, $K_2=\mathrm{diag}(k_{21},k_{22},\cdots,k_{2m})$.

It is customary that such a nonlinear function $\eta(C^T e,y)$ is said to belong to a sector $[K_1,K_2]$

3. Synchronization criteria

We first introduce the following definitions and lemmas.

Definition 1: [1, 2, 3] Let $V: R_+\times R^n\to R_+$, then V is said belong to class ϑ_0 if

(1) V is continuous in $(t_{k-1},t_k]\times R^n$ and for each $x\in R^n,\ k=1,2,\cdots,$

$$\lim_{(t,y)\to(t_k^+,x)} V(t,y)=V(t_k^+,x).$$

(2) V is locally Lipschitzian in x.

Definition 2: [1, 2, 3] For $(t,x)\in(t_{k-1},t_k]\times R^n$, we define

$$D^+V(t,x)=\lim_{s\to 0^+}\sup\frac{1}{s}[V(t+s,x(t+s))-V(t,x(t))].$$

Definition 3: [1, 2, 3] Comparison system: Let $V\in\vartheta_0$ and assume that

$$\begin{cases}D^+V(t,x)\leq g(t,V(t,x)), & t\neq t_k,\\ V(t,x+U(k,x))\leq \psi_k(V(t,x)), & t=t_k,\end{cases}$$

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where $g: R_+ \times R \to R$ is continuous in $(t_{k-1}, t_k] \times R$ and for each $x \in R$, $k = 1, 2, \cdots$,

$$\lim_{(t,y)\to(t_k^+,x)} g(t,y) = g(t_k^+, x)$$

exists. $\psi_k : R_+ \to R_+$ is no decreasing.

Then the following system

$$\begin{cases} \dot{\omega} = g(t,\omega), & t \neq t_k, \\ \omega(t_k^+) = \psi_k(\omega(t_k)), & t = t_k, \\ \omega(t_0^+) = \omega_0 \geq 0. \end{cases} \quad (9)$$

is the comparison system of (3).

Lemma 1: [1, 2, 3] Assume that the following three conditions are satisfied:

(1) $V: R_+ \times S_\rho \to R_+$, $\rho > 0$, $V \in \Omega_0$, $D^+V(t,x) \leq g(t, V(t,x)), t \neq t_k$;

(2) There exists a $\rho > 0$ such that $x \in S_{\rho_0}$ implies that $x + U(k,x) \in S_\rho$,

for all k and $V(t, x+U(k,x)) \leq \psi_k(V(t,x)), t = t_k, \; x \in S_{\rho_0}$;

(3) $\beta(\|x\|) \leq V(t,x) \leq \alpha(\|x\|)$ on $R_+ \times S_\rho$, where $\alpha(\|x\|), \beta(\|x\|) \in \aleph$.

Then the stability properties of the trivial solution of the comparison system (9) imply the corresponding stability properties of the trivial solution of the system (3).

Where $\aleph = \{\alpha(x) : \alpha \in C[R_+, R_+], \alpha(0) = 0,$ and $\alpha(x)$ is strictly increasing in $x\}$,

$S_\rho = \{x \in R^n : \|x\| < \rho, \|x\|$ denotes the Euclidean norm$\}$

Lemma 2: [1, 2, 3]

Let

$g(t,\omega) = \dot{\lambda}(t)\omega$, $\lambda \in C^1[R^+, R^+]$, $\dot{\lambda}(t) \geq 0$, $\psi_k(\omega) = d_k\omega$, $d_k \geq 0$ for all k, and $\sup_{1\leq k<\infty}\{d_k \exp[\lambda(t_{k+1}) - \lambda(t_k)]\} < \infty$.

Then the origin of the system (3) is global asymptotically stable if the conditions of Lemma 1 and the following conditions hold:

There exists a $\gamma > 1$ such that

$\lambda(t_{k+1}) + \ln(\gamma d_k) \leq \lambda(t_k)$, for all k.

There exists a $\gamma > 1$ such that

$\lambda(t_{2k+1}) + \ln(\gamma d_{2k} d_{2k-1}) \leq \lambda(t_{2k-1})$, for all k.

Now we use the following theorem to guarantee that our impulsive synchronization is asymptotically stable.

Theorem 1: Assume that the matrices $P > 0$, $Q > 0$, $\Lambda = diag(\lambda_1, \lambda_2, \cdots, \lambda_m) \geq 0$ and the numbers $a > 0$, $d_k \geq 0$ such that

$$\Omega = \begin{pmatrix} \Omega_{11} - aP & PB & PD + C(K_1 + K_2)\Lambda \\ * & -Q & 0 \\ * & * & -2\Lambda \end{pmatrix} < 0, \quad (10)$$

$$\|e(t_k) + I_k(e(t_k))\|^2 \leq \frac{d_k \lambda_{\min}(P)}{\lambda_{\max}(P) + \tau\lambda_{\max}(Q)} \|e(t_k)\|^2. \quad (11)$$

where $\Omega_{11} = (A-H)^T P + P(A-H) + Q - 2CK_1\Lambda K_2 C^T$.

If one of the following conditions is satisfied, then the origin of the error system (6) is global asymptotically stable, implying that the two systems (1) and (4) is impulsively synchronized.

(I) There exist a $\mu > 1$ such that

$0 \leq a(t_{k+1} - t_k) \leq -\ln(\mu d_k)$, for all k.

(II) There exist a $\mu > 1$ such that

$0 \leq a(t_{2k+1} - t_{2k-1}) \leq -\ln(\mu d_{2k} d_{2k-1})$, for all k.

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Proof: Consider the following Lyapunov function

$$V(e) = e^T Pe + \int_{t-\tau}^{t} e^T(s)Qe(s)ds .$$

Since $P>0, Q>0, b>0, c>0$ and inequality (7), we see that $V(e)$ is radically unbounded and

$$\lambda_{\min}(P)\|e\|^2 \le V(e) \le [\lambda_{\max}(P) + \tau\lambda_{\max}(Q)]\|e\|^2 , \quad (12)$$

when $t \ne t_k$, we have

$$\begin{aligned} D^+V(e) &= \dot{e}^T(t)Pe(t) + e^T(t)P\dot{e}(t) + e^T(t)Qe(t) - e^T(t-\tau)Qe(t-\tau) \\ &< a[e^T(t)Pe(t) + \int_{t-\tau}^{t} e^T(s)Qe(s)ds] \\ &= aV(e) . \end{aligned}$$

when $t = t_k$, by condition (12)(11), we have

$$\begin{aligned} V(e+I_k(e)) &= (e+I_k(e))^T P(e+I_k(e)) + \int_{t-\tau}^{t} (e+I_k(e))^T Q(e+I_k(e))ds \\ &\le d_k V(e) . \end{aligned}$$

Then we can obtain the following comparison system

$$\begin{cases} \dot{\omega}(t) = a\omega(t), & t \ne t_k, \\ \omega(t_k^+) = d_k\omega(t_k), & t = t_k, \\ \omega(t_0^+) = \omega_0 \ge 0. \end{cases}$$

To use Lemma 1, we shall prove that (2) of Lemma 1 holds. Let

$$\alpha(\|e\|) = [\lambda_{\max}(P) + \tau\lambda_{\max}(Q)]\|e\|^2 ,$$

$$\beta(\|e\|) = \lambda_{\min}(P)\|e\|^2 ,$$

$$\sigma = \sup_{1\le k<\infty}\{d_k \exp[a(t_{k+1} - t_k)]\} .$$

Since $\alpha(0) = 0$, then for any given ρ, there exists a $\rho_0 > 0$, such that

$$\alpha(\rho_0) < \frac{\beta(\rho)}{\sigma} .$$

From (12), we know that when $e \in S_{\rho_0}$, we have

$$V(e) \le \alpha(\rho_0) < \frac{\beta(\rho)}{\sigma} ,$$

It follows that

$$V(e + I_k(e)) \le d_k V(e) \le \sigma\alpha(\rho_0) < \beta(\rho) .$$

Using (12), we obtain

$$\beta(\|e + I_k(e)\|) \le V(e + I_k(e)) < \beta(\rho) .$$

so

$$\|e + I_k(e)\| < \rho .$$

This implies that

$$e + I_k(e) \in S_\rho .$$

Next, we can consider the conditions in Lemma 2. Since

$$\sup_{1\le k<\infty}\{d_k \exp[a(t_{k+1} - t_k)]\} \le d_k \exp[a\max\{t_1, t_2\}] < \infty ,$$

and

$$0 \le a(t_{k+1} - t_k) \le -\ln(\mu d_k) , \quad \mu > 1 ,$$

or

$$0 \le a(t_{2k+1} - t_{2k-1}) \le -\ln(\mu d_{2k} d_{2k-1}) , \quad \mu > 1 .$$

Thus the conditions in Lemma 2 are also satisfied, then the origin of the error system (6) is global asymptotically stable, implying that the two systems (1) and (4) is impulsively synchronized. This completes the proof of Theorem 1.

Remark 1: If we let $K_1 = 0$, the $\eta(C^T e, y)$ is said to belong to a sector $[0, K_2]$, the condition (7) becomes $\eta_i(\eta_i - k_{2i}C_i^T e) \le 0$.

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Remark 2: Similar to the impulsive control proposed in [2-4], a time invariant control can also be used.

Remark 3: In Theorem 1, the number a is independent of the matrices P, Q and the coefficient of system, thus Theorem 1 is less conservative than [4, Th1].

Remark 4: Let $\delta = \max\{t_1, t_2\}$, we can get the allowable bounds of impulsive intervals for the stable regions of the error system (6):

$$0 \le \delta \le -\frac{\ln(\mu d_k)}{a}.$$

4. Numerical examples

We consider the following Chua's oscillator

$$\begin{cases} \dot{x}_1 = \alpha(x_2 - x_1 - f(x_1)) \\ \dot{x}_2 = x_1 - x_2 + x_3 \\ \dot{x}_3 = -\beta x_2 - \gamma x_3 \end{cases},$$

with non-linear characteristic

$$f(x_1) = m_2 x_1 + 0.5(m_1 - m_2)(|x_1 + 1| - |x_1 - 1|),$$

and parameters $m_1 = -3$; $m_2 = -1$; $\alpha = 1$; $\beta = 2$; $\gamma = 1$. This system can be expressed in Lur'e form with

$$A = \begin{pmatrix} -\alpha & \alpha & 0 \\ 1 & -1 & 1 \\ 0 & -\beta & -\gamma \end{pmatrix} = \begin{pmatrix} -1 & 1 & 0 \\ 1 & -1 & 1 \\ 0 & -2 & -1 \end{pmatrix}; \quad B = 0; \quad D = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix};$$

$\varphi = -\alpha f(x_1)$ Belongs to sector $[1, 3]$.

Taking

$$I_k = \begin{pmatrix} -0.68 & 0 & 0 \\ 0 & -0.68 & 0 \\ 0 & 0 & -0.68 \end{pmatrix}; \quad \tau = 0; \quad a = 4;$$

$t_{2k+1} - t_{2k} = t_{2k} - t_{2k-1} = 0.04$;

Then

$$H = \begin{pmatrix} -6.3659 & 22.8599 & 57.3489 \\ -4.3924 & 2.0000 & -24.6400 \\ -7.6733 & 26.6400 & 2.0000 \end{pmatrix}.$$

$d_k = 0.7653$;

Since

$$0.1600 = a(t_{k+1} - t_k) \le -\ln(\mu d_k) = 0.1722.$$

Form Theorem 1, we know that impulsively controlled Chua's oscillator is synchronized.

5. Summary

The problem of synchronization criteria of impulsive Lurie systems with time-delay has been addressed. Based on stability theory of impulsive differential equation and new comparison theory of impulsive differential system, some simple yet less conservative criteria ensuring impulsive synchronization of the Lurie systems are derived. A numerical example is given to illustrate the effectiveness of the method.

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References

[1] Z.G. Li, C.Y. Wen, Y.C. Soh, IEEE Trans. Automat. Contr., 46 (2001) 894-903.

[2] J.T. Sun, Y.P. Zhang, L. Wang, Q.D. Wu, Physics, A30 (2006) 130-135.

[3] J. Zhou, X.H. Cheng, L. Xiang, Y.C. Zhang, Chaos, Solitons & Fractals, 33 (2007) 607-616.

[4] J.T. Sun, Y.P. Zhang, F. Qiao, Q.D. Wu, Chaos, Solitons & Fractals, 19 (2004) 1049-1055.

2013 © The American Computational Science Society. All rights reserved.

[5] B. Liu, G.R. Chen, K.L. Teo, X.Z. Liu, Chaos, Solitons & Fractals, 23 (2005) 1629-1641.

[6] B.J. Xu, X.Z. Liu, X.X. Liao, Computers and Mathematics with Applications, 47 (2004) 419-425.

[7] H.M. Guo, S.M. Zhong, Applied Mathematics and Computation, 191 (2007) 550-559.

[8] J.D. Cao, H.X. Li, D.W.C. Ho, Chaos, Solitons & Fractals, 23 (2005) 1285-1296.

[9] J. Cao, C. Yang, G. Zhou, F. Tang, P. Li, Discrete and Impulsive Systems, Series B: Applications & Algorithms, 14 (2007) 190-197.

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